Exam 3 Review

From Sec. 8.8

$(1-\alpha)100\%$ C.I. on μ

I) Any n; σ known; data is (approx.) Normal

$$\bar{X} \pm z\,(\sigma/\sqrt{n})$$

II) Small n; σ unknown; data is (approx.) Normal

$$\bar{X} \pm t\,(s/\sqrt{n})$$ where

t is from t-dist with df = n-1 using $\alpha/2$

t-distributions - properties

1) mound shape
2) $\mu = 0$
3) symmetric about $\mu = 0$
4) As n increases, $t \rightarrow z$
5) d.f.

III) Large n ($n \geq 30$) ; σ unknown

$$\bar{X} \pm Z\left(s/\sqrt{n}\right)$$

Sec. 8.9

$(1-\alpha)100\%$ C.I. on $\mu_X - \mu_Y$

I) $n < 20$, $m < 20$; X's are (approx.) Normal; Y's are (approx.) Normal

$$(\bar{X} - \bar{Y}) \pm t * s_p * \sqrt{\frac{1}{n} + \frac{1}{m}}$$ where

$$s_p = \sqrt{\frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}}$$ and

t from t-dist. with $df = n+m-2$ using $\alpha/2$

II) $n \geq 20$, $m \geq 20$

$$(\bar{X} - \bar{Y}) \pm z * \sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}$$

Note: This is called the "unpaired" difference in population means, or the difference in means of independent populations.

$(1-\alpha)100\%$ C.I. on $p_1 - p_2$

$\hat{p}_1 = \frac{x}{n}$ $\hat{p}_2 = \frac{y}{m}$

1) $n\hat{p}_1 > 5$
2) $n(1-\hat{p}_1) > 5$
3) $m\hat{p}_2 > 5$
4) $m(1-\hat{p}_2) > 5$

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n} + \frac{\hat{p}_2(1-\hat{p}_2)}{m}}$$

Sec. 8.10

$(1-\alpha)100\%$ CI on μ_d (paired diff.)

I) Small n

$$\bar{d} \pm t\,(s_d/\sqrt{n}) \quad \text{where}$$

$$s_d = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}} = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}}$$

t from t-dist with df = n-1 using $\alpha/2$

II) Large n

$$\bar{d} \pm z \left(s_d / \sqrt{n} \right)$$

ch 9 - Hypothesis Testing

Steps in Hyp. Test

1) State H_0
2) State H_A
3) Decide on α-level
4) Calculate test statistic
5) Find the p-value
6) Make decision
7) Conclusion (and Assumptions)

One-sided vs two-sided H_A

$H_0: \mu = \mu_0$ OR $H_0: \mu_d = 0$

$H_A: \mu > \mu_0$ $H_A: \mu_d > 0$

(small n, σ unknown
Normal data) (small n
Normal differences)

or

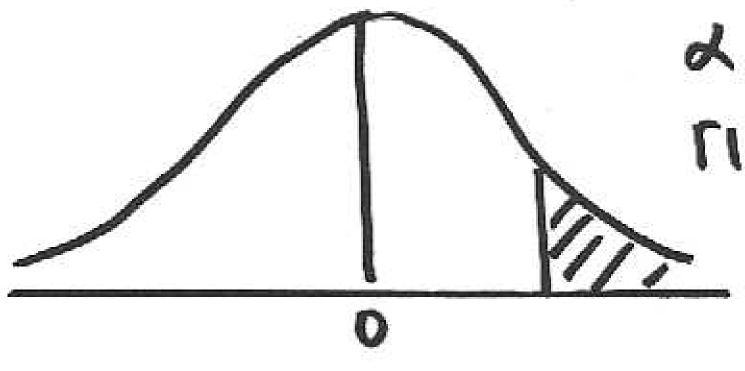
$H_0: \mu_X - \mu_Y = 0$

$H_A: \mu_X - \mu_Y > 0$

(Small m, n
Normal data)

p-value = $P(T \geq$ test.stat. $\mid H_0$ true$)$

Reject H_0 if p-value $\leq \alpha$



α area in right tail

$H_0: p = p_0$ OR $H_0: \mu = \mu_0$

$H_A: p > p_0$ $\quad$ $H_A: \mu > \mu_0$

(large n with σ unknown
or
any n with σ known and Normal data)

or

or

$H_0: \mu_d = 0$ $\quad$ $H_0: \mu_X - \mu_Y = 0$

$H_A: \mu_d > 0$ $\quad$ $H_A: \mu_X - \mu_Y > 0$

(large n) $\quad$ (large m, n)

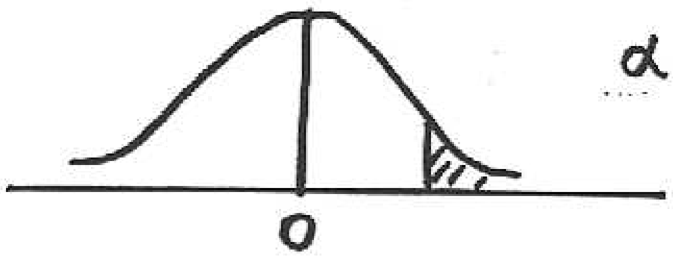
or

$H_0: p_1 - p_2 = 0$

$H_A: p_1 - p_2 > 0$

p-value = $P(Z \geq \text{test.stat.} \mid H_0 \text{ true})$

Reject H_0 if p-value $\leq \alpha$



α area in right tail

$H_0: \mu = \mu_0$ OR $H_0: \mu_d = 0$

$H_A: \mu < \mu_0$ $\quad$ $H_A: \mu_d < 0$

(small n, σ unknown
Normal data) $\quad$ (small n
Normal differences)

or

$H_0: \mu_X - \mu_Y = 0$

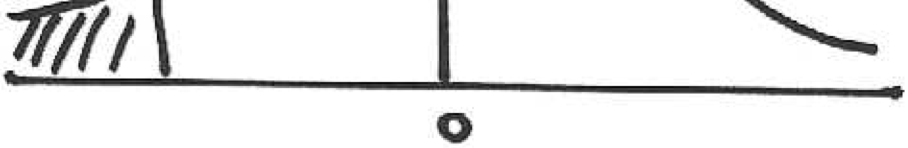
$H_A: \mu_X - \mu_Y < 0$

(Small m, n
Normal data)

p-value = $P(T \leq \text{test.stat.} \mid H_0 \text{ true})$

Reject H_0 if p-value $\leq \alpha$

α area in left tail



$H_0: p = p_0$ OR $H_0: \mu = \mu_0$

$H_A: p < p_0$ $\quad$ $H_A: \mu < \mu_0$

(large n with σ unknown
or
any n with σ known and
Normal data)

or

or

$H_0: \mu_d = 0$ $\quad$ $H_0: \mu_X - \mu_Y = 0$

$H_A: \mu_d < 0$ $\quad$ $H_A: \mu_X - \mu_Y < 0$

(large n) $\quad$ (large m, n)

or

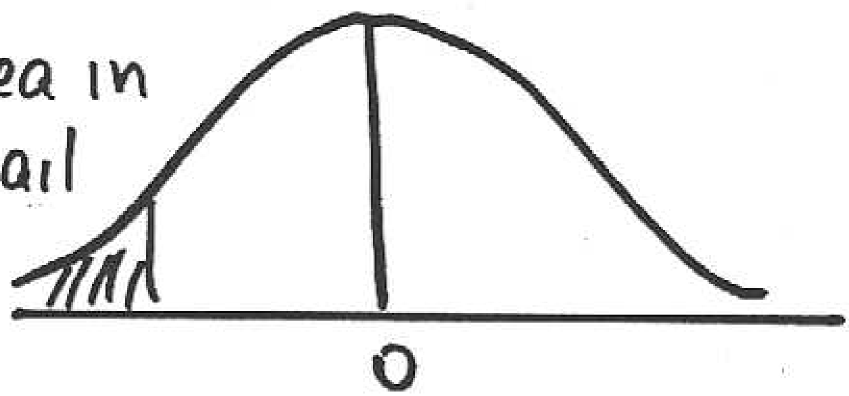
$H_0: p_1 - p_2 = 0$

$H_A: p_1 - p_2 < 0$

p-value $= P(Z \leq \text{test.stat.} \mid H_0 \text{ true})$

Reject H_0 if p-value $\leq \alpha$

α area in left tail



$H_0: \mu = \mu_0$
$H_A: \mu \neq \mu_0$
(small n, σ unknown
Normal Data)

or

$H_0: \mu_d = 0$
$H_A: \mu_d \neq 0$
(small n
Normal differences)

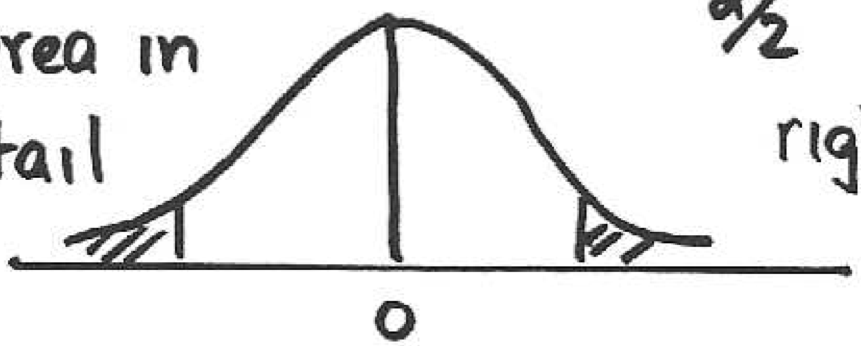
OR

$H_0: \mu_X - \mu_Y = 0$
$H_A: \mu_X - \mu_Y \neq 0$
(small m, n
Normal data)

p-value $= 2 * P(T \geq |\text{test.stat.}|)$

Reject H_0 if p-value $\leq \alpha$

$\alpha/2$ area in left tail



$\alpha/2$ area in right tail

$H_0: p = p_0$ OR $H_0: \mu = \mu_0$
$H_A: p \neq p_0$ $H_A: \mu \neq \mu_0$
(large n, σ unknown
or
any n, σ known with
Normal data)

OR

$H_0: \mu_d = 0$
$H_A: \mu_d \neq 0$
(large n)

OR

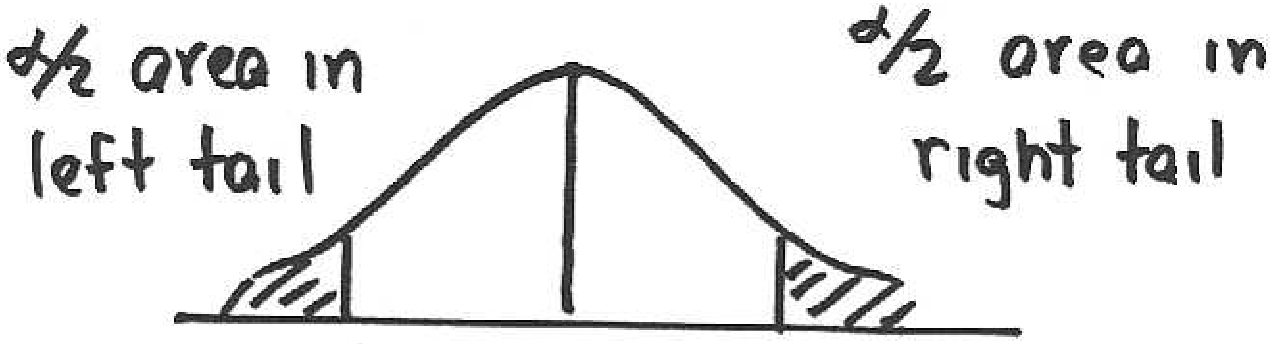
$H_0: \mu_X - \mu_Y = 0$
$H_A: \mu_X - \mu_Y \neq 0$
(large m, n)

OR

$H_0: p_1 - p_2 = 0$
$H_A: p_1 - p_2 \neq 0$

p-value $= 2 * P(Z \geq |\text{test.stat.}|)$

Reject H_0 if p-value $\leq \alpha$



Test Statistics - Forms

$H_0: \mu = \mu_0$

I) Small n; σ unknown; data approx. Normal

$$T = \frac{\bar{x} - \mu_0}{(s/\sqrt{n})} \qquad df = n-1$$

II) Large n; σ unknown

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

When n is large, find p-value using standard normal table (Table E)

III) Any n; σ known; data approx. Normal

$$Z = \frac{\bar{x} - \mu_0}{(\sigma/\sqrt{n})}$$

$H_0: \mu_d = 0$

I) Small n; differences approx. Normal

$$T = \frac{\bar{d} - \mu_d}{(S_d/\sqrt{n})} \qquad df = n-1$$

where $$S_d = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}} = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}}$$

II) Large n

$$Z = \frac{\bar{d} - \mu_d}{(S_d/\sqrt{n})}$$ where S_d as above

$H_0: \mu_X - \mu_Y = 0$

I) $n < 20$; $m < 20$; data approx. Normal

$$T = \frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \qquad df = n+m-2$$

where

$$S_p = \sqrt{\frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}}$$

II) $n \geq 20$; $m \geq 20$

$$Z = \frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}}$$

$H_0: p_1 - p_2 = 0$

1) $n\hat{p}_1 > 5$ $\qquad \hat{p}_1 = \frac{x}{n}$

2) $n(1-\hat{p}_1) > 5$

3) $m\hat{p}_2 > 5$ $\qquad \hat{p}_2 = \frac{y}{m}$

4) $m(1-\hat{p}_2) > 5$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n} + \frac{1}{m}\right)}}$$

where

$$\hat{p} = \frac{x + y}{n + m}$$

Hyp. Test - Equality of Variances

$H_0: \sigma_1^2 = \sigma_2^2$

$H_A: \sigma_1^2 \neq \sigma_2^2$

$$F = \frac{\text{bigger } s^2}{\text{smaller } s^2}$$