Exam 2 Review

ch5 - Expected Values

Def: The experimental expected value of a random variable X is the average value of X in a very very long sequence of replications of an experiment.

Def: For a discrete random variable X with probability distribution p(x) the theoretical expected value of X is

$$E(X) = \sum x\, p(x)$$

Note: This is often called the theoretical mean of r.v. X and is denoted by μ_X.

Law of Large Numbers for Independent Repetitions

Consider a random variable X which represents the outcome of an experiment. In a very long sequence of independent repetitions of the experiment, the average $\bar{X}$ of the observed values of X will be very close to the theoretical mean μ_X.

The Formula $\mu_x = np$

Suppose an event has probability p of occuring on a single trial, and n trials are conducted. Let r.v. X count the number of times the event of interest occurs in n trials. Then the expected value of x is

$$E(X) = \mu_x = np$$

Law of Large Numbers for Population Surveys

Suppose $X_1, X_2, \cdots, X_n$ are selected randomly w/o replacement from a very large pop. IF the sample size n is large, then the sample mean $\bar{x}$ of the observed values will be close to μ, the mean of the pop.

Box Models and Expected Values

Suppose r.v. X is the sum of repeated random draws from a box, then

$$E(X) = \mu_X = \text{box mean} * \#\text{draws}$$

In particular, if we make just one draw, then

$$E(X) = \mu_X = \text{box mean}$$

The formula above works whether we make our draws

with replacement or
w/o replacement.

Standard Deviation of a Random Variable

Def: Suppose r.v. X has prob. distribution $p(x)$. Then the (theoretical) variance of X is

$$\sigma_X^2 = \sum (x - \mu_X)^2 p(x)$$

and the (theoretical) standard deviation of X is

$$\sigma_X = \sqrt{\sum (x - \mu_X)^2 p(x)}$$

Box Models

Consider a box with N numbered balls. The box mean is

$$\mu_{box} = \frac{\Sigma x}{N}$$

and the box variance is

$$\sigma^2_{box} = \frac{\Sigma (x - \mu_{box})^2}{N}$$

and the box standard deviation is

$$\sigma_{box} = \sqrt{\frac{\Sigma (x - \mu_{box})^2}{N}}$$

Now, suppose X is the sum of n randomly selected balls from the box containing N balls. Then

$$\mu_X = n * \mu_{box}$$

If the balls are selected with replacement, then

$$\sigma_X = \sqrt{n} * \sigma_{box}$$

If the balls are selected w/o replacement, then

$$\sigma_X = \sqrt{n} * \sigma_{box} * \sqrt{\frac{N-n}{N-1}}$$

ch 6

Discrete Prob. Dist.

Probability Trees

Biomial Experiment:

1) Fixed number of trials, n
2) Only two possible outcomes for each trial (Succss and Failure)
3) p remains constont
4) Trials are indep.

Binomial Prob. Formula

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

for $x = 0, 1, 2, \ldots, n$

Binomial Dist:

$$\mu = np$$

$$\sigma = \sqrt{np(1-p)}$$

Binomial Prob. Table (G)

Poisson Dist.

Poisson Prob. Formula

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

for $x = 0, 1, 2, \cdots$

Poisson Dist:

$$\mu = m$$

$$\sigma = \sqrt{m}$$

Poisson Prob. Table (J)