The compound event

$E_1 \cap E_2$ = E_1 AND E_2

occurs if and only if both event E_1 occurs and event E_2

The compound event

$E_1 \cup E_2$ = E_1 OR E_2 or both

occurs if E_1 happens or
if E_2 happens or
if both events happen

Two events are mut. excl. if the occurence of one event precludes the occurence of the other event.

IF event E_1 and event E_2 are mut. excl. then

$$P(E_1 \text{ and } E_2) = 0$$

$$P(E_1 | E_2) = 0$$

$$P(E_2 | E_1) = 0$$

Gen. Add. Law ("OR")

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ AND } E_2)$$

Spec. Add. Law (Mut. excl. events)

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$$

Conditional Prob.

$P(E_2 \mid E_1)$

Gen. Mult. Law ("AND")

$P(E_1 \text{ AND } E_2) = P(E_1)P(E_2 \mid E_1)$
$= P(E_2)P(E_1 \mid E_2)$

Spec. Mult. Law (Ind. Events)

$P(E_1 \text{ and } E_2) = P(E_1)P(E_2)$

Two events are independent if the occurence of one event does not affect the prob. of the other event.

$$P(E_1) = P(E_1 | E_2)$$

and

$$P(E_2) = P(E_2 | E_1)$$

Conditional Prob. Formulas

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

SIMULATION

Simulation methods (or Monte Carlo simulation) are techiques which use a computer to perform an experiment a large number of times.

Five Step Simulation Method

1) Choose a Probability Model
2) Define one simulation
3) Define event of interest
4) Repeat simulation n times
5) Compute experimental prob.

Simulating Random Sampling via a Box Model

Box Modeling is a method of simulating data by randomly drawing a number from a box repeatedly, either with or w/o replacement. The box may contain any number of real numbers, some (or all) of which may appear in the box more than once.

Random Sampling - With or W/O Replacement

When sampling <u>with</u> rep<u>lacement</u> entails replacing the selected ball before drawing the next selection.

When sampling w<u>ithout</u> rep<u>lace</u>ment, the selected ball is <u>not</u> replaced before the next selection.

Multiplication Rule for Multi-Stage Experiments

Suppose an experiment is conducted in K stages. Stage 1 can be performed in any of N_1 ways; stage 2 can be performed in any of N_2 ways; etc. Then there are

$$N_1 * N_2 * \cdots * N_K$$

ways of performing the experiment

ch 5 EXPECTED VALUES

Def: The experimental expected value of a random variable X is the average value of X in a very very long sequence of replications of an experiment.

Def: For a discrete random variable X with probability distribution p(x) the theoretical expected value of X is

$$E(X) = \sum x\, p(x)$$

Note: This is often called the theoretical mean of r.v. X and is denoted by μ_X.

Law of Large Numbers for Independent Repetitions

Consider a random variable X which represents the outcome of an experiment. In a very long sequence of independent repetitions of the experiment, the average $\bar{X}$ of the observed values of X will be very close to the theoretical mean μ_X.

The Formula $\mu_x = np$

Suppose an event has probability p of occuring on a single trial, and n trials are conducted. Let r.v. X count the number of times the event of interest occurs in n trials. Then the expected value of X is

$$E(X) = \mu_x = np$$

Law of Large Numbers for Population Surveys

Suppose $X_1, X_2, \cdots, X_n$ are selected randomly w/o replacement from a very large pop. If the sample size n is large, then the sample mean $\bar{x}$ of the observed values will be close to μ, the mean of the pop.

Box Models and Expected Values

Suppose r.v. X is the sum of repeated random draws from a box, then

$E(x) = \mu_x =$ box mean * #draws

In particular, if we make just one draw, then

$E(x) = \mu_x =$ box mean

Ex: P. 227 Text

The formula above works whether we make our draws

with replacement or
w/o replacement.

Standard Deviation of a Random Variable

Def: Suppose r.v. X has prob. distribution $p(x)$. Then the (theoretical) variance of X is

$$\sigma_X^2 = \sum (x - \mu_X)^2 p(x)$$

and the (theoretical) standard deviation of X is

$$\sigma_X = \sqrt{\sum (x - \mu_X)^2 p(x)}$$

Box Models

Consider a box with N numbered balls. The box mean is

$$\mu_{box} = \frac{\Sigma x}{N}$$

and the box variance is

$$\sigma^2_{box} = \frac{\Sigma (x - \mu_{box})^2}{N}$$

and the box standard deviation is

$$\sigma_{box} = \sqrt{\frac{\Sigma (x - \mu_{box})^2}{N}}$$

Now, suppose X is the sum of n randomly selected balls from the box containing N balls. Then

$$\mu_X = n * \mu_{box}$$

If the balls are selected with replacement, then

$$\sigma_X = \sqrt{n} * \sigma_{box}$$

If the balls are selected w/o replacement, then

$$\sigma_X = \sqrt{n} * \sigma_{box} * \sqrt{\frac{N-n}{N-1}}$$