Lecture notes on Experiment Design & Data Analysis

Design of Engineering Experiments Part 3 – Analysis of Variance (6)

- Comparing more that two factor levels…**the analysis of variance**
  - ANOVA decomposition of total variability
  - Statistical testing & analysis
  - Checking assumptions, model validity
  - Post-ANOVA testing of means
  - Box-Cox method
- **Sample size** determination

Power and Sample Size Determination

- In any experimental design problem, a critical decision is the choice of sample size – that is, determine the number of replicates to run.
- Sample size is usually determined by the trade-off between
  - Statistical considerations such as Power of tests, and precision of estimations; and
  - the availability of resources such as money, time, man power, etc.
  - In general, if the experimenter is interested in detecting small effects, more replicates are required than if the experimenter is interested in detecting large effects.

Power and Sample Size Determination Fixed Effects Case

- Can choose the sample size to detect a specific difference in means and achieve desired values of **type I and type II errors**
- Type I error – reject H_0 when it is true (α)
- Type II error – fail to reject H_0 when it is false (β)
- **Power** = 1 - β

Power and Sample Size Determination Fixed Effects Case

- β=1-P{Reject H_0|H_0 is false} =1-P{F_0>$F_{\alpha,a-1,N-a}$ | H_0 is false}
- To evaluate β, we need to know the distribution of the test statistic F_0 if the null hypothesis is false.
- It can be shown that, if H_0 is false, F_0 is distributed as a **noncentral F** distribution with *a-1* and *N-a* degrees of freedom and the noncentrality parameter Φ (measuring the deviation from H_0). The bigger the Φ, the further away from H_0 and the higher the power.

$$\Phi = \frac{1}{\sigma}\sqrt{\frac{\sum_{i=1}^{a} n\tau_i^2}{a}}$$

Power and Sample Size Determination Fixed Effects Case

- The power of a test depends on
  - the signifiance level : the larger α , the higher power;
  - how much deviation is from H_0.
    - the strength of signal: the higher signal to noise ratio, the higher power;
    - sample size: the larger sample size, the higher power

$$\Phi = \frac{1}{\sigma}\sqrt{\frac{\sum_{i=1}^{a} n\tau_i^2}{a}} \Rightarrow \Phi^2 = \frac{n\sum_{i=1}^{a}\tau_i^2}{a\sigma^2}$$

Sample Size Determination
Fixed Effects Case

- **Operating characteristic curves** plot β against Φ.
- Goal: find the *smallest* sample size needed to achieve
  - a (predetermined) power
  - with *at most* a (predetermined) type I error rate
  - for *at least* a (predetermined) signal
- Why? On one hand, we want sample size to be large enough to detect important deviations from H_0 with high probability. However, on the other hand, the sample size should not be unnecessary large such that the cost of the study is too high AND practically insignificant deviations from H_0 becomes statistical significant with a high probability

Sample Size Determination Fixed Effects Case---use of OC Curves

- The **OC curves** for the fixed effects model are in the Appendix, Table V, pg. 613
- A very common way to use these charts is to define a difference in two means *D* when the null hypothesis should be rejected, then the minimum value of Φ^2 is

$$\Phi^2 = \frac{nD^2}{2a\sigma^2}$$

- Typically work in term of the ratio of D/σ and try values of *n* until the **desired power** is achieved
- R will perform power and sample size calculations
- There are some other methods discussed in the text

Example 3-10

- Want $\beta \leq 0.1$, D=75, α=0.01 , assume σ=25.

$$\Phi^2 = \frac{n(75)^2}{2(4)(25^2)} = 1.125n$$

n	Φ^2	a(n-1)	Power (1- β)
4	4.5	4(3)=12	0.65
5	5.625	4(4)=16	0.8
6	6.75	4(5)=20	0.9

R code

Power and Sample size calculation

```
#In R, go to
#Packages -> Install package(s) -> find "usa CA1"--> highlight it and click ok-->
#find "pwr" in the pop-up window and highlight-> click ok

library(pwr)

#k=Number of groups, n Number of observations (per group)
#f=square root (D^2/(2*k*sigma^2))
#sig.level=Significance level (Type I error probability)
#power=Power of test (1 minus Type II error probability)

#obtain sample size n given k, f,alpha and power
pwr.anova.test(k=4,f=sqrt(1.125),sig.level=0.01,power=0.9)

#obtain the actual power given k,f,n=6 and alpha
pwr.anova.test(k=4,f=sqrt(1.125),n=6,sig.level=0.01)
```

R code and output

```
>pwr.anova.test(k=4,f=sqrt(1.125),,sig.level=0.01,power=0.9)

     Balanced one-way analysis of variance power calculation

              k = 4
              n = 5.8107
              f = 1.060660
      sig.level = 0.01
          power = 0.9
      NOTE: n is number in each group

> pwr.anova.test(k=4,f=sqrt(1.125),n=6,sig.level=0.01,)

     Balanced one-way analysis of variance power calculation

              k = 4
              n = 6
              f = 1.060660
      sig.level = 0.01
          power = 0.915384
 NOTE: n is number in each group
```