Design of Engineering Experiments Part 6 – Blocking & Confounding in the 2^k

- Text reference, Chapter 7
- **Blocking** is a technique for dealing with controllable **nuisance** variables
- Two cases are considered
  - Replicated designs
  - Unreplicated designs

Blocking a Replicated Design

- This is the same scenario discussed previously (Chapter 5, Section 5-6)
- If there are *n* replicates of the design, then each replicate is a block
- Each **replicate** is run in one of the **blocks** (time periods, batches of raw material, etc.)
- Runs within the block are **randomized**

Chemical Process Example- chapter6 page 204

Study the effect of the reactant concentration (A) and the amount of the catalyst (B) on the conversion (yield) of a chemical process.

Run	*A*	*B*	y_{i1}	y_{i2}	y_{i3}	$\bar{y}_{i.}$
1	−	−	28.00	25.00	27.00	26.67
2	+	−	36.00	32.00	32.00	33.33
3	−	+	18.00	19.00	23.00	20.00
4	+	+	31.00	30.00	29.00	30.00

A = reactant concentration, *B* = catalyst amount, *y* = recovery

Blocking a Replicated Design

Consider the example from Section 6-2; $k = 2$ factors, $n = 3$ replicates

This is the "usual" method for calculating a block sum of squares

Table 7-1 Chemical Process Experiment in Three Blocks

	Block 1	Block 2	Block 3
	$(1) = 28$	$(1) = 25$	$(1) = 27$
	$a = 36$	$a = 32$	$a = 32$
	$b = 18$	$b = 19$	$b = 23$
	$ab = 31$	$ab = 30$	$ab = 29$
Block totals:	$B_1 = 113$	$B_2 = 106$	$B_3 = 111$

$$SS_{Blocks} = \sum_{i=1}^{3} \frac{B_i^2}{4} - \frac{y_{...}^2}{12}$$
$$= 6.50$$

ANOVA for the Blocked Design
Page 267

Table 7-2 Analysis of Variance for the Chemical Process Experiment in Three Blocks

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
Blocks	6.50	2	3.25		
A (concentration)	208.33	1	208.33	50.32	0.0004
B (catalyst)	75.00	1	75.00	18.12	0.0053
AB	8.33	1	8.33	2.01	0.2060
Error	24.84	6	4.14		
Total	323.00	11			

Confounding in Blocks

- Now consider the **unreplicated** case
- Clearly the previous discussion does not apply, since there is only one replicate
- Sometime, it is impossible to perform a complete replicate of a factorial design in one block.
- **Confounding** is a design technique for arranging a complete factorial experiment in blocks, where the block size is smaller than the number of treatment combinations in one replicate.

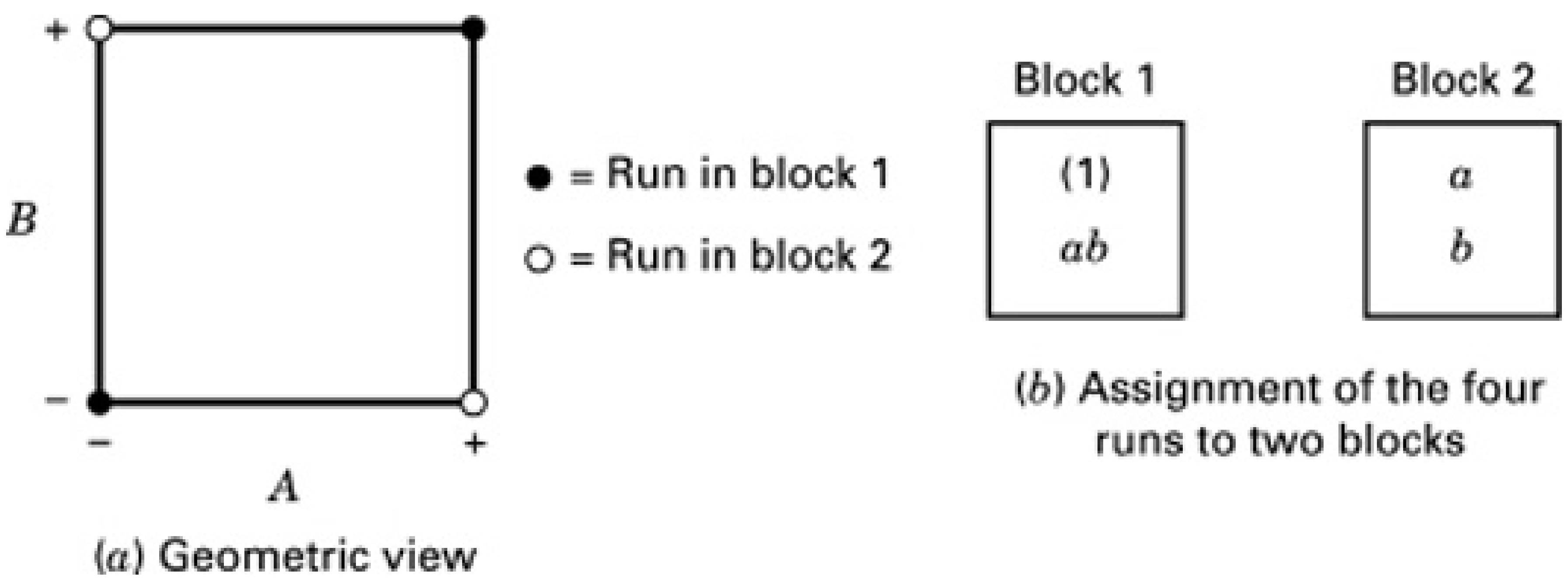


Figure 7.1 (p. 267)

A 2^2 design in two blocks.

Table 7-3 Table of Plus and Minus Signs for the 2^2 Design

Treatment Combination	Factorial Effect				
	I	*A*	*B*	*AB*	Block
(1)	+	−	−	+	2
a	+	+	−	−	1
b	+	−	+	−	1
ab	+	+	+	+	2

Table 7.3 (p. 268)

Table of Plus and Minus Signs for the 2^2 Design

Table 7-4 Table of Plus and Minus Signs for the 2^3 Design

Treatment Combination	Factorial Effect								
	I	*A*	*B*	*AB*	*C*	*AC*	*BC*	*ABC*	Block
(1)	+	−	−	+	−	+	+	−	1
a	+	+	−	−	−	−	+	+	2
b	+	−	+	−	−	+	−	+	2
ab	+	+	+	+	−	−	−	−	1
c	+	−	−	+	+	−	−	+	2
ac	+	+	−	−	+	+	−	−	1
bc	+	−	+	−	+	−	+	−	1
abc	+	+	+	+	+	+	+	+	2

Table 7.4 (p. 268)

Table of Plus and Minus Signs for the 2^3 Design

Experiment from Example 6-2

Table 6-10 Pilot Plant Filtration Rate Experiment

Run Number	Factor A	B	C	D	Run Label	Filtration Rate (gal/h)
1	—	−	−	−	(1)	45
2	+	−	−	−	*a*	71
3	−	+	−	−	*b*	48
4	+	+	−	−	*ab*	65
5	−	−	+	−	*c*	68
6	+	−	+	−	*ac*	60
7	−	+	+	−	*bc*	80
8	+	+	+	−	*abc*	65
9	−	−	−	+	*d*	43
10	+	−	−	+	*ad*	100
11	−	+	−	+	*bd*	45
12	+	+	−	+	*abd*	104
13	−	−	+	+	*cd*	75
14	+	−	+	+	*acd*	86
15	−	+	+	+	*bcd*	70
16	+	+	+	+	*abcd*	96

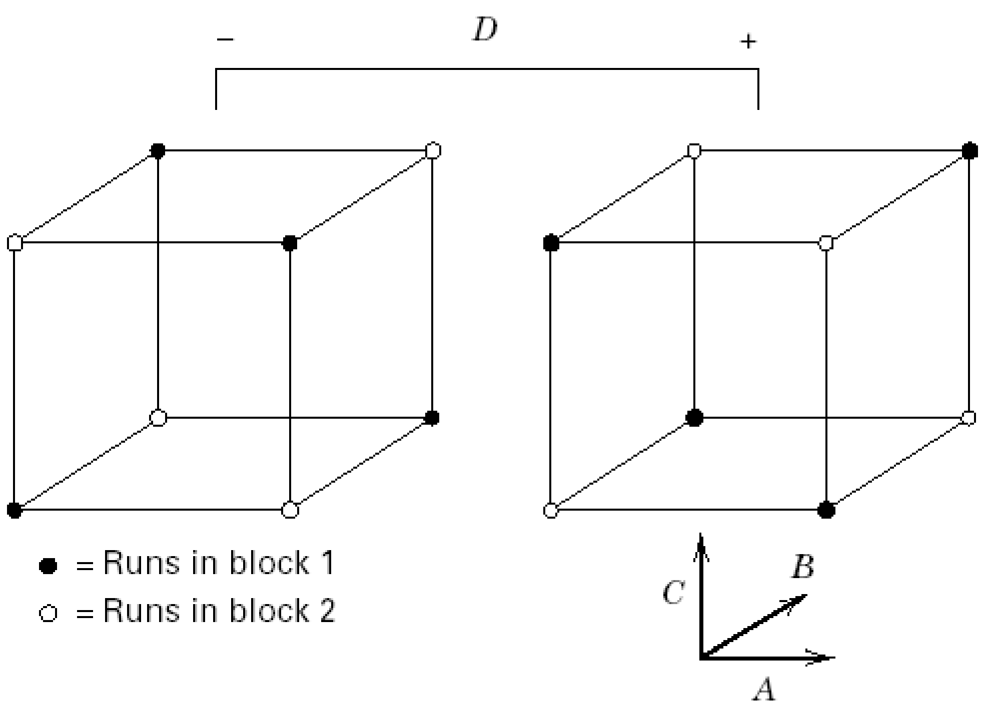
Suppose only 8 runs can be made from one batch of raw material

The Table of + & - Signs, Example 6-4

Table 6-11 Contrast Constants for the 2^4 Design

	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	$ABCD$
(1)	−	−	+	−	+	+	−	−	+	+	−	+	−	−	+
a	+	−	−	−	−	+	+	−	−	+	+	+	+	−	−
b	−	+	−	−	+	−	+	−	+	−	+	+	−	+	−
ab	+	+	+	−	−	−	−	−	−	−	−	+	+	+	+
c	−	−	+	+	−	−	+	−	+	+	−	−	+	+	−
ac	+	−	−	+	+	−	−	−	−	+	+	−	−	+	+
bc	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+
abc	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
d	−	−	+	−	+	+	−	+	−	−	+	−	+	+	−
ad	+	−	−	−	−	+	+	+	+	−	−	−	−	+	+
bd	−	+	−	−	+	−	+	+	−	+	−	−	+	−	+
abd	+	+	+	−	−	−	−	+	+	+	+	−	−	−	−
cd	−	−	+	+	−	−	+	+	−	−	+	+	−	−	+
acd	+	−	−	+	+	−	−	+	+	−	−	+	+	−	−
bcd	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−
abcd	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

ABCD is Confounded with Blocks (Page 272)



(a) Geometric view

Block 1	Block 2
(1) = 25	a = 71
ab = 45	b = 48
ac = 40	c = 68
bc = 60	d = 43
ad = 80	abc = 65
bd = 25	bcd = 70
cd = 55	acd = 86
$abcd$ = 76	abd = 104

(b) Assignment of the 16 runs to two blocks

Figure 7-4 The 2^4 design in two blocks for Example 7-2.

•The order in which the treatment combinations are run within a block is randomly determined.

•Which block to run first is also randomly decided

Effect Estimates

Table 7-6 Effect Estimates for the Blocked 2^4 Design in Example 7-2

Model Term	Regression Coefficient	Effect Estimate	Sum of Squares	Percent Contribution
A	10.81	21.625	1870.5625	26.30
B	1.56	3.125	39.0625	0.55
C	4.94	9.875	390.0625	5.49
D	7.31	14.625	855.5625	12.03
AB	0.062	0.125	0.0625	<0.01
AC	−9.06	−18.125	1314.0625	18.48
AD	8.31	16.625	1105.5625	15.55
BC	1.19	2.375	22.5625	0.32
BD	−0.19	−0.375	0.5625	<0.01
CD	−0.56	−1.125	5.0625	0.07
ABC	0.94	1.875	14.0625	0.20
ABD	2.06	4.125	68.0625	0.96
ACD	−0.81	−1.625	10.5625	0.15
BCD	−1.31	−2.625	27.5625	0.39
Block (*ABCD*)		−18.625	1387.5625	19.51

- In the original experiment (Example 6-2), ABCD=1.375.
- In the present example, ABCD=-18.625.
  - ABCD is confounded with blocks,

    block effect=-18.625-1.375=-20
  - If the experiment had not been run in blocks, and if an effect of magnitude -20 had affected the first 8 trials (which would have been selected in a random fashion, because the 16 trials would be run in random order in an unblocked design), the result could have been very different

The ANOVA

Table 7-7 Analysis of Variance for Example 7-2

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
Blocks (*ABCD*)	1387.5625	1			
A	1870.5625	1	1870.5625	89.76	<0.0001
C	390.0625	1	390.0625	18.72	0.0019
D	855.5625	1	855.5625	41.05	0.0001
AC	1314.0625	1	1314.0625	63.05	<0.0001
AD	1105.5625	1	1105.5625	53.05	<0.0001
Error	187.5625	9	20.8403		
Total	7111.4375	15			

The *ABCD* interaction (or the block effect) is not considered as part of the error term

The reset of the analysis is unchanged from Example 6-2

Example 6-2 (from lecture 16)

Analysis of Variance Table

Response: FRate

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
A	1	1870.6	1870.6	83.368	1.67e-05
C	1	390.1	390.1	17.384	0.003124
D	1	855.6	855.6	38.131	0.000267
A:C	1	1314.1	1314.1	58.565	6.00e-05
A:D	1	1105.6	1105.6	49.273	0.000110
C:D	1	5.1	5.1	0.226	0.647483
A:C:D	1	10.6	10.6	0.471	0.512032
Residuals	8	179.5	22.4		

Another Illustration of the Importance of Blocking

Table 7-8 The Modified Data from Example 7-2

Run Order	Std. Order	Factor *A*: Temperature	Factor *B*: Pressure	Factor *C*: Concentration	Factor *D*: Stirring Rate	Response Filtration Rate
8	1	−1	−1	−1	−1	25
11	2	1	−1	−1	−1	71
1	3	−1	1	−1	−1	28
3	4	1	1	−1	−1	45
9	5	−1	−1	1	−1	68
12	6	1	−1	1	−1	60
2	7	−1	1	1	−1	60
13	8	1	1	1	−1	65
7	9	−1	−1	−1	1	23
6	10	1	−1	−1	1	80
16	11	−1	1	−1	1	45
5	12	1	1	−1	1	84
14	13	−1	−1	1	1	75
15	14	1	−1	1	1	86
10	15	−1	1	1	1	70
4	16	1	1	1	1	76

Now the first eight runs (in run order) have filtration rate reduced by 20 units

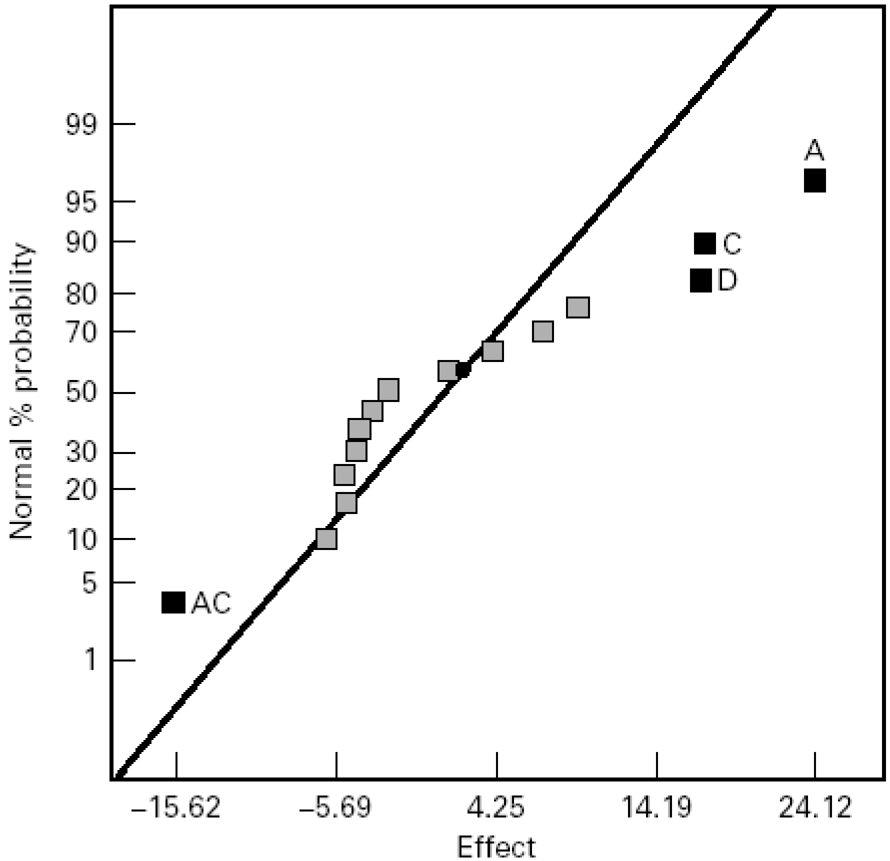


Figure 7-5 Normal probability plot for the data in Table 7-8.

The interpretation is harder; not as easy to identify the large effects

One important interaction is not identified (*AD*)

Failing to block when we should have causes problems in interpretation the result of an experiment and can mask the presence of real factor effects

Confounding in Blocks

- More than two blocks (page 275)
  - The two-level factorial can be confounded in 2, 4, 8, … (2^p, $p > 1$) blocks
  - For **four** blocks, select **two** effects to confound, automatically confounding a **third** effect
  - See example, page 275
  - Choice of confounding schemes non-trivial; see Table 7-9, page 277
- Partial confounding (page 278)

General Advice About Blocking

- When in doubt, block
- Block out the nuisance variables you know about, randomize as much as possible and rely on randomization to help balance out unknown nuisance effects
- Measure the nuisance factors you know about but can’t control (ANCOVA)
- It may be a good idea to conduct the experiment in blocks even if there isn't an obvious nuisance factor, just to protect against the loss of data or situations where the complete experiment can’t be finished