Design of Engineering Experiments Part 4 – Introduction to Factorials (2)

- Text reference, Chapter 5
- **General principles** of factorial experiments
- The **two-factor factorial** with fixed effects
- The **ANOVA** for factorials
- Extensions to more than two factors
- **Quantitative** and **qualitative** factors – response curves and surfaces

Choice of Sample Size

If the difference in any two row means is D, then

$$\Phi^2 = \frac{nbD^2}{2a\sigma^2},$$

If the difference in any two column means is D, then

$$\Phi^2 = \frac{nbD^2}{2a\sigma^2}$$

If the difference between any two interaction effects is

$$\Phi^2 = \frac{nD^2}{2\sigma^2[(a-1)(b-1)+1]}$$

- Suppose that before running the experiment we decide that the null hypothesis should be rejected with a high probability 0.9 if the difference in mean battery life between any two temperatures is as great as 40 hours and if we assume that the standard deviation of battery life is approximately 25. Assuming α=0.05

```
>Phi<-(3*40^2)/(2*3*25^2)
>pwr.anova.test(k=4,f=sqrt(Phi),sig.level=0.05,power=0.9)
```

- Balanced one-way analysis of variance power calculation
- k = 4
- n = 3.911752
- f = 1.131371
- sig.level = 0.05
- power = 0.9

- NOTE: n is number in each group

Factorials with More Than Two Factors

- Basic procedure is similar to the two-factor case; all *abc…kn* treatment combinations are run in random order
- ANOVA identity is also similar:

$$SS_T = SS_A + SS_B + \cdots + SS_{AB} + SS_{AC} + \cdots$$
$$+ SS_{ABC} + \cdots + SS_{AB \cdots K} + SS_E$$

Table 5-12 The Analysis of Variance Table for the Three-Factor Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Expected Mean Square	F_0
A	SS_A	$a-1$	MS_A	$\sigma + \frac{bcn\sum \tau_i^2}{a-1}$	$F_0 = \frac{MS_A}{MS_E}$
B	SS_B	$b-1$	MS_B	$\sigma^2 + \frac{acn\sum \beta_j^2}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
C	SS_C	$c-1$	MS_C	$\sigma^2 + \frac{abn\sum \gamma_k^2}{c-1}$	$F_0 = \frac{MS_C}{MS_E}$
AB	SS_{AB}	$(a-1)(b-1)$	MS_{AB}	$\sigma^2 + \frac{cn\sum\sum (\tau\beta)_{ij}^2}{(a-1)(b-1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
AC	SS_{AC}	$(a-1)(c-1)$	MS_{AC}	$\sigma^2 + \frac{bn\sum\sum (\tau\gamma)_{ik}^2}{(a-1)(c-1)}$	$F_0 = \frac{MS_{AC}}{MS_E}$
BC	SS_{BC}	$(b-1)(c-1)$	MS_{BC}	$\sigma^2 + \frac{an\sum\sum (\beta\gamma)_{jk}^2}{(b-1)(c-1)}$	$F_0 = \frac{MS_{BC}}{MS_E}$
ABC	SS_{ABC}	$(a-1)(b-1)(c-1)$	MS_{ABC}	$\sigma^2 + \frac{n\sum\sum\sum (\tau\beta\gamma)_{ijk}^2}{(a-1)(b-1)(c-1)}$	$F_0 = \frac{MS_{ABC}}{MS_E}$
Error	SS_E	$abc(n-1)$	MS_E	σ^2	
Total	SS_T	$abcn-1$			

Example 5-3

- A soft drink bottler is interested in obtaining more uniform fill heights in the bottles produced by his manufacturing process. The filling machine theoretically fills each bottle to the correct target height, but in practice, there is variation around this target and the possible factors are: A- the percent carbonation, B- the operating pressure in the filler, C-the bottles produced per minute or the line speed (C).

Table 5-13 Fill Height Deviation Data for Example 5-3

	Operating Pressure (B)								
	25 psi				30 psi				
	Line Speed (C)				Line Speed (C)				
Percent Carbonation (A)	200		250		200		250		$y_{i...}$
10	−3, −1	(−4)	−1, 0	(−1)	−1, 0	(−1)	1, 1	(2)	−4
12	0, 1	(1)	2, 1	(3)	2, 3	(5)	6, 5	(11)	20
14	5, 4	(9)	7, 6	(13)	7, 9	(16)	10, 11	(21)	59
$B \times C$ Totals $y_{.jk.}$	6		15		20		34		$75 = y_{....}$
$y_{.j..}$	21				54				

$A \times B$ Totals

$y_{ij..}$

A	B = 25	30
10	−5	1
12	4	16
14	22	37

$A \times C$ Totals

$y_{i.k.}$

A	C = 200	250
10	−5	1
12	6	14
14	25	34

```
> #Fitting the 3-way ANOVA model
> softdrink.fit<-lm(height~A*B*C)
> anova(softdrink.fit)
```

Analysis of Variance Table

Response: height

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
A	2	252.750	126.375	178.4118	1.186e-09	***
B	1	45.375	45.375	64.0588	3.742e-06	***
C	1	22.042	22.042	31.1176	0.0001202	***
A:B	2	5.250	2.625	3.7059	0.0558081	.
A:C	2	0.583	0.292	0.4118	0.6714939	
B:C	1	1.042	1.042	1.4706	0.2485867	
A:B:C	2	1.083	0.542	0.7647	0.4868711	
Residuals	12	8.500	0.708			

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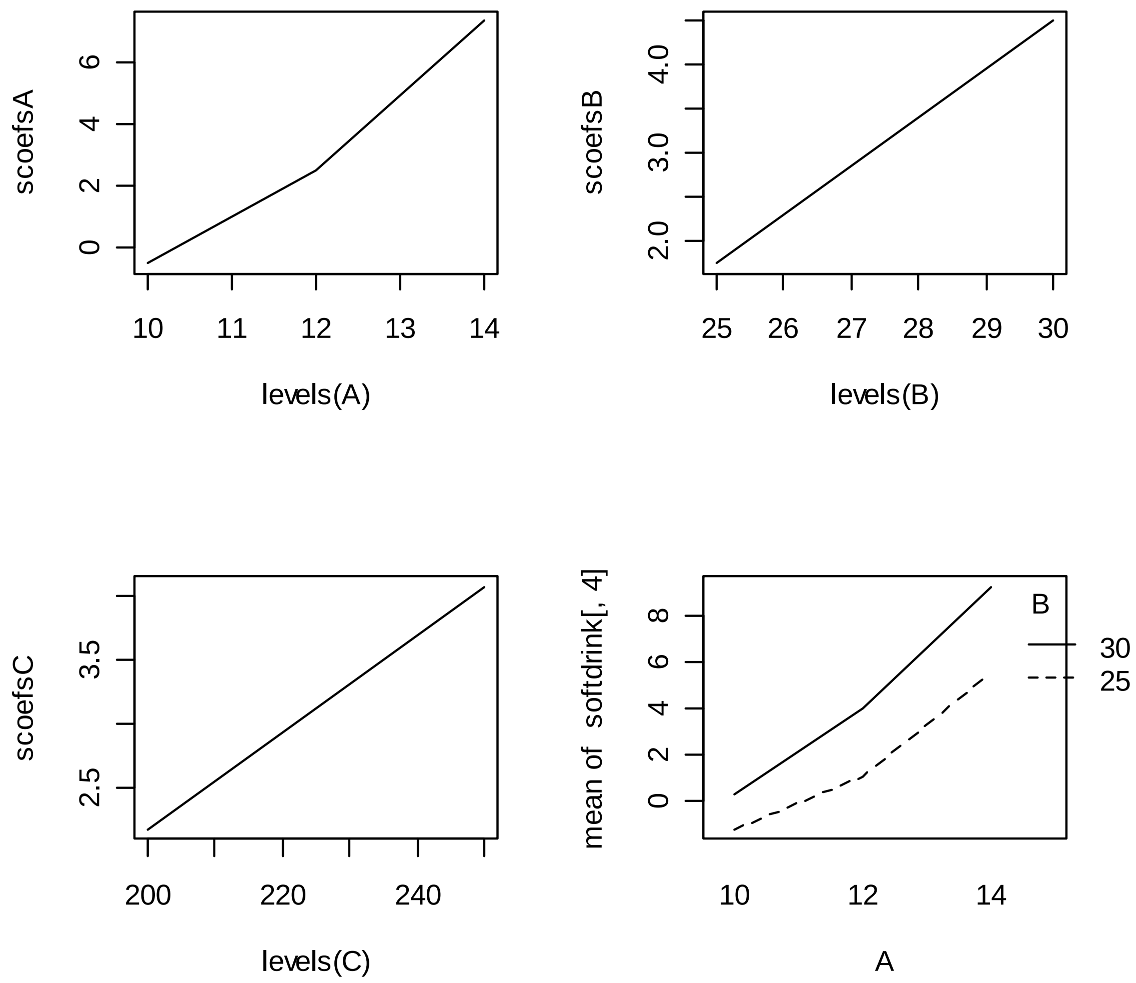
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


scoefsA
0
2
4
6
10
11
12
13
14
levels(A)
scoefsB
2.0
3.0
4.0
25
26
27
28
29
30
levels(B)
scoefsC
2.5
3.5
200
220
240
levels(C)
mean of softdrink[, 4]
0
2
4
6
8
10
12
14
A
B
30
25

Quantitative and Qualitative Factors

- The basic ANOVA procedure treats every factor as if it were **qualitative**
- Sometimes an experiment will involve both **quantitative** and **qualitative** factors, such as in Example 5-1
- This can be accounted for in the analysis to produce **regression models** for the quantitative factors at each level (or combination of levels) of the qualitative factors
- These **response curves** and/or **response surfaces** are often a considerable aid in practical interpretation of the results

Blocking In a Factorial Design

- Example 5-6

An engineer is studying methods for improving the ability to detect targets on a radar scope. Two factors she considers to be important are the amount of background noise (ground clutter) on the scope and the type of filter placed over the screen. The experiment will be conducted by operator. Because of operator availability, it is convenient to select an operator and keep him or her at the scope until all the necessary runs have been made. Furthermore, operators differ in their skill and ability to use the scope.

Table 5-19 Intensity Level at Target Detection

Operators (blocks)	1		2		3		4	
Filter Type	1	2	1	2	1	2	1	2
Ground clutter								
Low	90	86	96	84	100	92	92	81
Medium	102	87	106	90	105	97	96	80
High	114	93	112	91	108	95	98	83

Table 5-18 Analysis of Variance for a Two-Factor Factorial in a Randomized Complete Block

Source of Variation	Sum of Squares	Degrees of Freedom	Expected Mean Square	F_0
Blocks	$\frac{1}{ab}\sum_k y_{..k}^2 - \frac{y_{...}^2}{abn}$	$n-1$	$\sigma^2 + ab\sigma_\delta^2$	
A	$\frac{1}{bn}\sum_i y_{i..}^2 - \frac{y_{...}^2}{abn}$	$a-1$	$\sigma^2 + \frac{bn\sum \tau_i^2}{a-1}$	$\frac{MS_A}{MS_E}$
B	$\frac{1}{an}\sum_j y_{.j.}^2 - \frac{y_{...}^2}{abn}$	$b-1$	$\sigma^2 + \frac{an\sum \beta_j^2}{b-1}$	$\frac{MS_B}{MS_E}$
AB	$\frac{1}{n}\sum_i\sum_j y_{ij.}^2 - \frac{y_{...}^2}{abn} - SS_A - SS_B$	$(a-1)(b-1)$	$\sigma^2 + \frac{n\sum\sum(\tau\beta)_{ij}^2}{(a-1)(b-1)}$	$\frac{MS_{AB}}{MS_E}$
Error	Subtraction	$(ab-1)(n-1)$	σ^2	
Total	$\sum_i\sum_j\sum_k y_{ijk}^2 - \frac{y_{...}^2}{abn}$	$abn-1$		

```
> #Fitting the 3-way ANOVA model
> target.fit<-lm(intensity~clutter*type+operator)
> anova(target.fit)
Analysis of Variance Table

Response: intensity
              Df  Sum Sq Mean Sq F value    Pr(>F)
clutter        2  335.58  167.79  15.1315 0.0002527 ***
type           1 1066.67 1066.67  96.1924 6.447e-08 ***
operator       3  402.17  134.06  12.0892 0.0002771 ***
clutter:type   2   77.08   38.54   3.4757 0.0575066 .
Residuals     15  166.33   11.09
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```