Design of Engineering Experiments Part 4 – The Blocking Principle

- Text Reference, Chapter 4
- **Blocking** and **nuisance factors**
- The randomized complete block design or the **RCBD**
- Extension of the ANOVA to the RCBD
- Other blocking scenarios…Latin square designs

The Blocking Principle

- **Blocking** is a technique for dealing with **nuisance factors**
- A **nuisance** factor is a factor that probably has some effect on the response, but it's of no interest to the experimenter…however, the variability it transmits to the response needs to be minimized
- Typical nuisance factors include batches of raw material, operators, pieces of test equipment, time (shifts, days, etc.), different experimental units
- **Many** industrial experiments involve blocking (or should)
- Failure to block is a common flaw in designing an experiment (consequences?)

Block what you can and Randomize what you can't

- If the nuisance variable is **known** and **controllable**, we use **blocking**
- If the nuisance factor is **known** and **uncontrollable**, sometimes we can use the **analysis of covariance** (see Chapter 15) to remove the effect of the nuisance factor from the analysis
- If the nuisance factor is **unknown** and **uncontrollable** (a **"lurking" variable)**, we hope that **randomization** balances out its impact across the experiment
- Sometimes several sources of variability are **combined** in a block, so the block becomes an aggregate variable

A reseacher wants to study the effect of the type of fertilizer on barley yield. She is interested in three types of fertilizer (A, B, C) and has three plots of land (1, 2, 3) to grow the barley. An experiment was run in which fertilizers A, B, and C were applied to the barley in plots 1, 2, and 3 respectively. The mean yield (kilograms) from each fertilizer/plot combination are pictured below:

PLOT	FERTILIZER	MEAN YIELD
1	A	14
2	B	15
3	C	13

It appears that B may be the fertilizer to produce the highest yield. In reality, suppose the use of fertilizer A will on average produce 5 kilo. more than B and 7 kilo. more than C when growing conditions are identical.

- Question: What could have happened in this experiment to produce such a low mean yield for fertilizer A?
- Answer: The growing conditions of plot 1 may be much poorer than the growing conditions of plots 2 and 3. This would reduce the mean yield for A since it was the only fertilizer used on plot 1 and lead to very misleading results due to a poor experimental design.
- Note: Any conclusion regarding the effect of using a fertilizer is completely *confounded* with the growing condition effects of the plots.
- Question: How could you improve the design so this confounding is removed?

Answer: Use each fertilizer on each plot. This is done by first dividing each plot into three equal size subplots:

PLOT	SUBPLOT	SUBPLOT	SUBPLOT
1	1	2	3
2	1	2	3
3	1	2	3

Next, the three fertilizers are randomly assigned to the three subplots for each plot (each fertilizer is applied to each plot once).

PLOT	FERTILIZER	FERTILIZER	FERTILIZER
1	B	C	A
2	C	B	A
3	C	A	B

Then the data will consist of three mean yield measurements for each fertilizer.

PLOT	MN YIELD	MN YIELD	MN YIELD
1	9 (B)	7 (C)	14 (A)
2	14 (C)	14 (B)	21 (A)
3	12 (C)	19 (A)	16 (B)

If we summarize the data into the following table, we can see what the effect of growing condition had on the results:

		PLOT 1	2	3	MEAN
FERT.	A	14	21	19	$\bar{X}_A = 18$
	B	9	14	16	$\bar{X}_B = 13$
	C	7	14	12	$\bar{X}_C = 11$

Note the lower yield on plot 1 for each fertilizer. Because each fertilizer was applied to each plot, we retain a balance that enables us to study differences in yield due to fertilizers in the presence of variable growing conditions.

Hardness Testing Experiment (page 49)

Specimen	Tip 1	Tip2	diff (d_j)
1	7	6	1
2	3	3	0
3	3	5	-2
4	4	3	1
5	8	8	0
6	3	3	0
7	2	4	-2
8	9	9	0
9	5	4	1
10	4	5	-1
Mean(Y)	4.8	4.9	-0.1
SD(Y)	2.39	2.2	1.197

By pairing, the estimate of variability is reduced by nearly 50%

A completely randomized design for this problem will

- increase the variability of the hardness measurements
- tend to inflate the experimental error
- make a true difference harder to detect

It is beneficial if two hardness determination can be made on the same specimen

The Hardness Testing Example

- Text reference, pg 120
- We wish to determine whether 4 different tips produce different (mean) hardness reading on a Rockwell hardness tester
- Assignment of the tips to an **experimental unit**; that is, a test coupon
- Structure of a completely randomized experiment
- The test coupons are a source of **nuisance variability**
- Alternatively, the experimenter may want to test the tips across coupons of various hardness levels
- The need for blocking

The Hardness Testing Example

- To conduct this experiment as a RCBD, assign all 4 tips to each coupon
- Each coupon is called a "**block**"; that is, it's a more homogenous experimental unit on which to test the tips
- Variability **between** blocks can be large, variability **within** a block should be relatively small
- In general, a **block** is a specific level of the nuisance factor
- A complete replicate of the basic experiment is conducted in each block
- A block represents a **restriction on randomization**
- All runs **within** a block are **randomized**

The Hardness Testing Example

- Suppose that we use $b = 4$ blocks:

Table 4-1 Randomized Complete Block Design for the Hardness Testing Experiment

Test Coupon (Block)			
1	2	3	4
Tip 3	Tip 3	Tip 2	Tip 1
Tip 1	Tip 4	Tip 1	Tip 4
Tip 4	Tip 2	Tip 3	Tip 2
Tip 2	Tip 1	Tip 4	Tip 3

- Notice the **two-way structure** of the experiment
- Once again, we are interested in testing the equality of treatment means, but now we have to remove the variability associated with the nuisance factor (the blocks)

Extension of the ANOVA to the RCBD

- Suppose that there are *a* treatments (factor levels) and *b* blocks
- A **statistical model** (effects model) for the RCBD is

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \begin{cases} i = 1, 2, ..., a \\ j = 1, 2, ..., b \end{cases}$$

- The relevant (fixed effects) hypotheses are

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_a \text{ where } \mu_i = (1/b)\sum_{j=1}^{b}(\mu + \tau_i + \beta_j) = \mu + \tau_i$$

Extension of the ANOVA to the RCBD

ANOVA partitioning of total variability:

$$\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij}-\bar{y}_{..})^2 = \sum_{i=1}^{a}\sum_{j=1}^{b}[(\bar{y}_{i.}-\bar{y}_{..})+(\bar{y}_{.j}-\bar{y}_{..})$$

$$+(y_{ij}-\bar{y}_{i.}-\bar{y}_{.j}+\bar{y}_{..})]^2$$

$$= b\sum_{i=1}^{a}(\bar{y}_{i.}-\bar{y}_{..})^2 + a\sum_{j=1}^{b}(\bar{y}_{.j}-\bar{y}_{..})^2$$

$$+\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij}-\bar{y}_{i.}-\bar{y}_{.j}+\bar{y}_{..})^2$$

$$SS_T = SS_{Treatments} + SS_{Blocks} + SS_E$$

Extension of the ANOVA to the RCBD

The degrees of freedom for the sums of squares in

$$SS_T = SS_{Treatments} + SS_{Blocks} + SS_E$$

are as follows:

$$ab - 1 = a - 1 + b - 1 + (a - 1)(b - 1)$$

$$E\left(\frac{SS_{Treatment}}{a-1}\right)=\sigma^2+\frac{b\sum_{i=1}^{a}\tau_i^2}{a-1},$$

$$E\left(\frac{SS_{Block}}{b-1}\right)=\sigma^2+\frac{a\sum_{j=1}^{b}\beta_j^2}{b-1}$$

$$E\left(\frac{SSE}{(a-1)(b-1)}\right)=\sigma^2$$

ANOVA Display for the RCBD

Table 4-2 Analysis of Variance for a Randomized Complete Block Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}}$	$a-1$	$\frac{SS_{\text{Treatments}}}{a-1}$	$\frac{MS_{\text{Treatments}}}{MS_E}$
Blocks	SS_{Blocks}	$b-1$	$\frac{SS_{\text{Blocks}}}{b-1}$	
Error	SS_E	$(a-1)(b-1)$	$\frac{SS_E}{(a-1)(b-1)}$	
Total	SS_T	$N-1$		

$$F_0 \overset{\text{under } H_0}{\sim} F(a-1,(a-1)(b-1))$$

Is this a one tail test or two tail test?

Manual computing…see Equations (4-9) – (4-12), page 124

Vascular Graft Example (pg. 124)

- Suspect the extrusion pressure affects the occurrence of flicks on the Vascular Graft device.
- To conduct this experiment as a RCBD, assign all 4 pressures to each of the 6 batches of resin
- Each batch of resin is called a "**block**"; that is, it's a more homogenous experimental unit on which to test the extrusion pressures

Table 4-3 Randomized Complete Block Design for the Vascular Graft Experiment

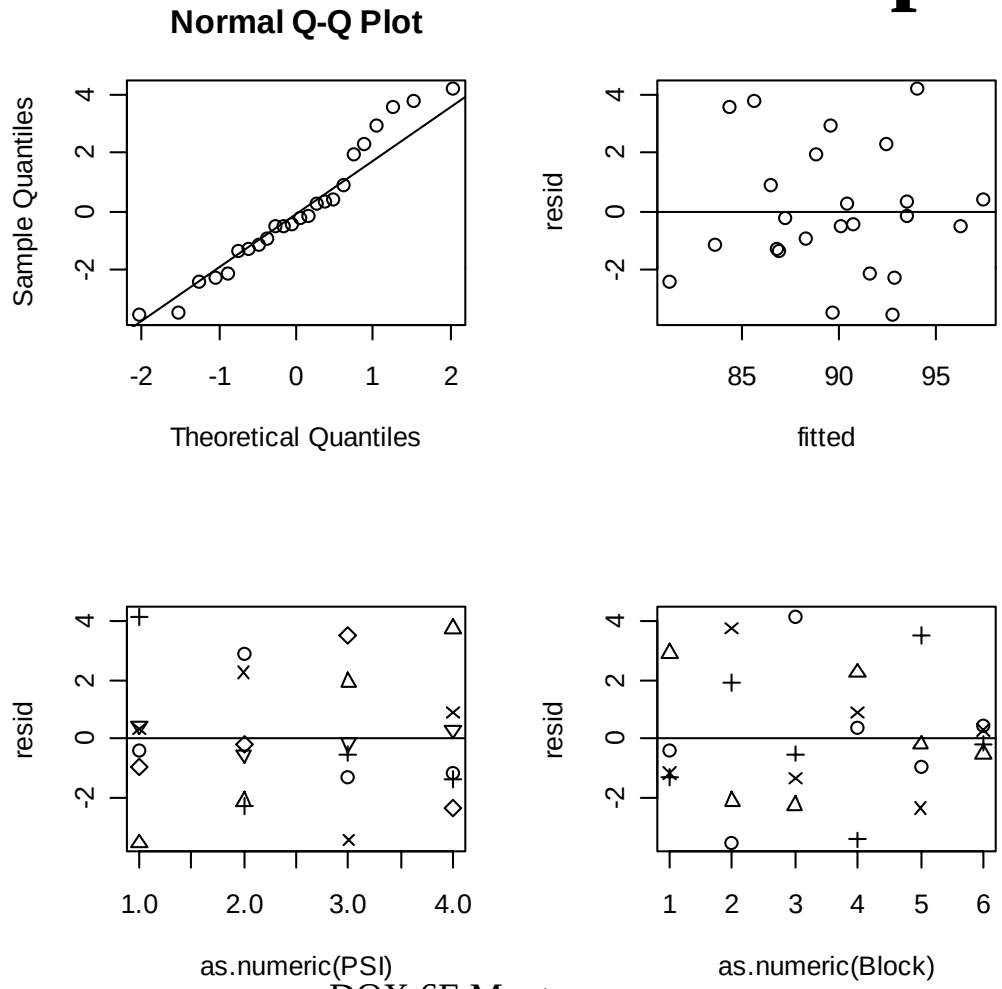
Extrusion Pressure (PSI)	Batch of Resin (Block) 1	2	3	4	5	6	Treatment Total
8500	90.3	89.2	98.2	93.9	87.4	97.9	556.9
8700	92.5	89.5	90.6	94.7	87.0	95.8	550.1
8900	85.5	90.8	89.6	86.2	88.0	93.4	533.5
9100	82.5	89.5	85.6	87.4	78.9	90.7	514.6
Block Totals	350.8	359.0	364.0	362.2	341.3	377.8	$y_{..} = 2155.1$

Vascular Graft Example

```
resin=read.table("Resin.txt", header=T)
attach(resin)
Block<-factor(Block)
PSI<-factor(PSI)
#Fitting the RCBD model
resin.fit<-lm(Flick~PSI+Block)
anova(resin.fit)
```

```
Analysis of Variance Table
Response: Flick
          Df  Sum Sq Mean Sq F value   Pr(>F)
PSI        3 178.171  59.390  8.1071 0.001916 **
Block      5 192.252  38.450  5.2487 0.005532 **
Residuals 15 109.886   7.326
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual Analysis for the Vascular Graft Example



Residual Analysis for the Vascular Graft Example

- Basic residual plots indicate that **normality**, **constant variance** assumptions are satisfied
- No obvious problems with **randomization**
- **No patterns in the residuals vs. block**
- Can also plot residuals versus the pressure (residuals by factor)
- These plots provide more information about the constant variance assumption, possible outliers

Multiple Comparisons for the Vascular Graft Example – Which Pressure is Different?

```
>TukeyHSD(aov(resin.fit),"PSI",ordered=TRUE)

Tukey multiple comparisons of means
   95% family-wise confidence level
   factor levels have been ordered
Fit: aov(formula = resin.fit)
$PSI
                   diff       lwr                upr     p adj
8900-9100 3.150000 -1.353828  7.653828 0.2257674
8700-9100 5.916667  1.412839 10.420495 0.0086667
8500-9100 7.050000  2.546172 11.553828 0.0020883
8700-8900 2.766667 -1.737161  7.270495 0.3245644
8500-8900 3.900000 -0.603828  8.403828 0.1013084
8500-8700 1.133333 -3.370495  5.637161 0.8854831
```


Note: output in Figure 4-2 is based on LSD

Other Aspects of the RCBD
See Text, Section 4-1.3, pg. 130

- The RCBD utilizes an **additive model** – no interaction between treatments and blocks
- Treatments and/or blocks as random effects
- Missing values
- What are the **consequences** of **not blocking** if we should have?
- **Sample sizing** in the RCBD? The **OC curve** approach can be used to determine the number of blocks to run..see page 131