Stat 101 Review Problems for Midterm II

To be discussed in review session Wed. Nov. 19, 8-10PM Physics 203

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1. **Vegetarian meals.** An airline is looking at how many vegetarian meals they should plan on for a transatlantic flight. Normally they plan on 15 meals for the 350 passengers on the Airbus A330. For years, using this strategy, they ran out of meals only 10% of the time (so 90% of the time all vegetarians were happy), but since they switched caterers two months ago, they have run out in 17 of their 100 flights. They wonder if this increase is real, or just due to bad luck. If it's real, they will start ordering more vegetarian meals.

a) What are the null and alternative hypotheses?

b) Test the null hypothesis at $\alpha = .05$. (Always include a discussion of assumptions and conditions). What do you conclude (in plain English)? Show all your work.

c) (4 points) Give a 95% confidence interval for the true proportion of times they have run out of vegetarian meals since the switch.

d) (2 points) Suppose that a year later you find out that the proportion had, in fact, *not* increased from 10%. Did you make an error in a)? If so, what kind?

2. **Alzheimer's Evaluation.** Screening for Alzheimer's disease is time consuming and costly. Recently Solomon *et al* have proposed a series of short tests to use instead of a lengthy battery. The examination consists of a series of 6 short tests at the end of which a patient will be categorized as having Alzheimer's (and needing further testing), or being clear of Alzheimer's. Past experience shows that 75% of healthy people pass all the tests, while none of the patients with Alzheimer's pass all 6. Here is a chart showing the percentages of Healthy and Sick patients passing different numbers of tests:

Number of tests failed	0	1	2	3	4	5	6
Healthy people	75%	19%	2%	2%	1%	1%	0%
Alzheimer's patients	0%	10%	70%	5%	4%	1%	10%

The table above indicates, for example, that 19% of all healthy patients and 10% of all sick patients fail exactly 1 test.

We could view this as a hypothesis-testing situation with the hypotheses:

H_0: Patient is healthy
H_a: Patient has Alzheimer's

Using these hypotheses:

a) (2 points) What would a type I error mean *in this context*?

b) (2 points) What would a type II error mean *in this context*?

c) (3 points) Up until now, they have been diagnosing some one as having Alzheimer's (needing further testing) if they fail *any* of the tests. For this rule, what is the type I error of this test? The type II error? Does this seem like a reasonable trade off?

d) (3 points) Suppose they want a type I error no bigger than 5%. What number of tests would a patient have to fail in order to be classified as having Alzheimer's? What level of α did they actually use?

e) (3 points) What is the power of the rule in d)?

f) (3 points) Another psychiatrist points out that by increasing α just 2%, a different cut off rule can be used that will increase the power substantially. Explain.

3. **Probability Models**

a) Newborn infants are born male and female in approximately equal proportions. Assuming that the probability does not depend on previous births, which of the following sequences is most likely:

1. BBBGGG
2. BBBBBB
3. BGGBGB
4. GGGGGG
5. None of the above

Explain your answer briefly.

b) People arrive at my store at an average rate of 10 per hour. Their arrivals are independent of each other and of the time of day. What probability model might reasonably model the arrivals?

1. Poisson
2. Negative Binomial
3. Hypergeometric
4. Geometric
5. Normal
6. None of the above

Explain your answer briefly.

c) For the model in b) what is the probability that no one comes to my store in the next 6 minutes?

4. **Bag of coins.** Suppose nine coins are placed in a jar. Three of these coins have heads on both sides, four have tails on both sides and the rest are fair.

   a) If I pick a coin at random and then flip it, what is the probability of getting tails ? (Hint: use a tree – this will be helpful in b) as well)

   b) Suppose I flipped heads. What is the probability that the other side is also heads?

   c) Suppose I pick one coin out of the jar, and without looking at it, want to decide if it's the two headed coin. I set up the hypotheses:

      H_0: The coin is two-headed vs. H_A: The coin is not two-headed

      I flip the coin once and decide to reject H_0 if it the coin comes up tails.

      1) How large are α and β for this test?

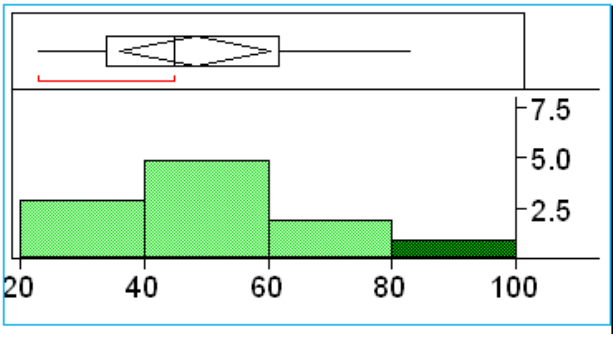
      2) How might you reasonably reduce the probability of a type I error?

5. **Killer bats.** An experiment was performed to answer the question: how far apart are a bat and its prey when the bat first senses the victim's presence? Using very elaborate equipment, it was possible to measure this distance on 11 occasions. The data were: 62, 52, 68, 23, 34, 45, 27, 42, 83, 56 and 40 cm.

Some summary statistics and graphics from JMP follow:

Moments

Mean	48.36364
Std Dev	18.08465
Std Error Mean	5.45273
N	11.00000
Sum Weights	11.00000



a. Construct a 95% confidence interval for the true mean distance from a victim that a bat can first sense.

b. Test the hypothesis that the mean distance is 40 cm.

c. What assumptions did you make about the data in parts (a) and (b)? Do they seem reasonable?

d. How would a 99% confidence interval for the true distance differ from your answer in (a)? Explain briefly.

e. How large would the sample size have to be to get a confidence interval half as wide as the one you got in (a), at the same confidence level?