More Tutorial at www.dumblittledoctor.com

(10 pts) **Problem 1.** For each of the following relations, please specify whether or not they form a function. For those which are a function, please state the domain.

a. $\{(2,3),(3,5),(3,6),(4,9)\}$

(*) Not a function. $x = 3$ has two corresponding y values.

b. $y^2 - x = 0$

(*) Not a function. To see this, solve for y:

$$\begin{aligned} y^2 - x &= 0 \\ y^2 &= x \\ y &= \pm\sqrt{x} \end{aligned}$$

Because y corresponds to both $\sqrt{x}$ and $-\sqrt{x}$, this is not a function.

c. $y = \frac{1}{x^2-1}$

(*) This is a function. The domain is any real number such that the denominator is non-zero. So to find what is *not* in the domain, solve the denominator for zero.

$$\begin{aligned} x^2 - 1 &= 0 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

So if x is either 1 or -1, then the denominator is zero. So the domain is $\{x \mid x \neq \pm 1\}$.

(15 pts) **Problem 2.** Let the function f be defined as follows: $f(x) = x^2 + 6x$.

a. For what value(s) of x is $f(x) = 16$?

(*)

$$\begin{aligned} f(x) &= 16 \\ x^2 + 6x &= 16 \\ x^2 + 6x - 16 &= 0 \\ (x+8)(x-2) &= 0 \end{aligned}$$

$$x + 8 = 0 \quad x - 2 = 0$$

$$x = -8 \quad x = 2$$

b. Please compute the difference quotient for $f(x)$.

(*) Recall the difference quotient is $\frac{f(x+h)-f(x)}{h}$.

First let us find $f(x+h)$.
$f(x+h) = (x+h)^2 + 6(x+h) = (x+h)(x+h) + 6(x+h) = x^2 + 2hx + h^2 + 6x + 6h$.

Now we see

$$\begin{aligned} \tfrac{f(x+h)-f(x)}{h} &= \tfrac{x^2+2hx+h^2+6x+6h-(x^2+6x)}{h} \\ &= \tfrac{x^2+2hx+h^2+6x+6h-x^2-6x}{h} \\ &= \tfrac{2hx+h^2+6h}{h} \\ &= \tfrac{h(2x+h+6)}{h} \\ &= 2x + h + 6 \end{aligned}$$

More Tutorial at www.dumblittledoctor.com

(15 pts) **Problem 3.** Let the function f be defined as follows: $f(x) = x^2 + 2x + 1$.

a. What is the average rate of change from 1 to 3?

(*) Average rate of change is given by the formula $\frac{f(b)-f(a)}{b-a}$

$$\begin{aligned} \frac{f(b)-f(a)}{b-a} &= \frac{f(3)-f(1)}{3-1} \\ &= \frac{16-4}{2} \\ &= 6 \end{aligned}$$

b. What is the *equation* of the secant line from 1 to 3?

(*) A line has the form $y = mx + b$. We need to find m and b.

m, the slope of the secant line, is equivalent to the average rate of change. So m has already been computed in the previous question.

So far, then, I know $y = 6x + b$. To find b, I will take the coordinates of a point on the line, plug it into my equation, and solve for b.

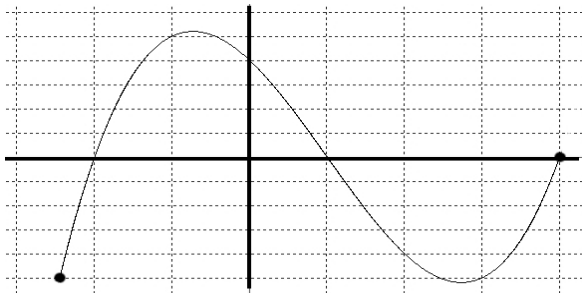
When $x = 1$, $y = f(1) = 4$. So I let $x = 1$ and $y = 4$ in my equation for the secant line:

$$\begin{aligned} y &= 6x + b \\ 4 &= 6(1) + b \\ -2 &= b \end{aligned}$$

Then the equation is $y = 6x - 2$.

More Tutorial at www.dumblittledoctor.com

(15 pts) **Problem** 4. Let the function f be that whose graph is depicted to the right.



a. Please state the x-intercepts.

(*) $x = -2, 1, 4$.

b. Please state the y-intercept.

(*) $y = 4$.

c. For what value(s) of x is $f(x) = 4$?

(*) $x = -1.5, 0$.

d. What is the value of $f(3)$?

(*) $f(3) = -5$.

e. Please state the domain.

(*) $[-2.5, 4]$.

More Tutorial at www.dumblittledoctor.com

(10 pts) **Problem 5.** Let the function f be that whose graph is depicted to the right.

a. For what interval(s) is $f(x)$ increasing?

(*) $(-1, 2) \cup (4, 5)$

b. Please list the coordinate(s) of the relative maxima.

(*) $(2, 4)$.

More Tutorial at www.dumblittledoctor.com

(15 pts) **Problem 6.** The functions $S(p)$ and $D(p)$ model supply and demand, respectively, of quantities of *Watsonbucks* coffee.

$$S(p) = 10p - 20 \qquad D(p) = 40 - 5p$$

a. Please compute the equilibrium price. That is, the price for which supply equals demand.

(*)

$$\begin{aligned} S(p) &= D(p) \\ 10p - 20 &= 40 - 5p \\ 15p - 20 &= 40 \\ 15p &= 60 \\ p &= 4 \end{aligned}$$

b. What is the quantity supplied at the equilibrium price?

(*) $S(4) = 10(4) - 20 = 20$.

More Tutorial at www.dumblittledoctor.com

(10 pts) **Problem 7.** Let the function f be defined as below:

$$f(x) = \begin{cases} 2 & \text{if } x < 3; \\ 2x - 4 & \text{if } 3 \leq x < 6; \\ -x & \text{if } x \geq 6. \end{cases}$$

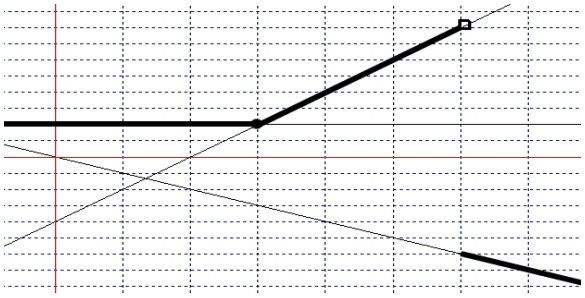
a. Please state the value of $f(1)$.

(*) Note that $x = 1$ is in the first partition. So $f(1) = 2$.

b. Please state the value of $f(6)$.

(*) Note that $x = 6$ is in the third partition. So $f(6) = -6$.

c. Please sketch a graph of the $f(x)$. Be sure to include all three partitions in your graph.



(10 pts) **Problem 8.** Let the function f be defined as follows: $f(x) = x^3$.

a. Please state the equation and show a graph of $f(x)$ after shifting right by 3 units, up by 2 units, and reflecting about the x-axis.

(*) $-(x-3)^3 - 2$

b. Please state the equation and show a graph of $f(x)$ after shifting left by 3 units and compressing vertically by a factor of $\frac{1}{2}$.

(*) $\frac{1}{2}(x+3)^3$