

Exam

1. 18pts) For each of the limits in part a)-c), determine whether the limit exists as a number, as an infinite limit, or does not exist. If the limit is a number, evaluate it. Give reasons.
 - a) $\lim_{x \rightarrow 0} \frac{x}{\sin x + \sin^2(x)}$
 - b) $\lim_{x \rightarrow \infty} \sqrt{x+3} - \sqrt{x-3}$
 - c) $\lim_{x \rightarrow 2} \frac{|(x-2)(x+1)|}{x-2}$
2. 12pts) Determine if the functions in part a)-b) is continuous on the given interval.
 - a) $f(x) = \ln(\sqrt{x^2 - x - 2})$ on $[0, 2]$
 - b) $f(x) = \tan(x^2)$ on $[0, 1]$
3. 12pts) Let $f(x) = x + \cos x + 1$.
 - a). Show that f has at least one zero.
 - b) You want to use the Newton's method to find a root of the function. Let $c_0 = 0$, find c_1
4. 12pts) Consider the equation $y + x \sin y = e^x$. If y is a function of x , calculate y' and y'' , the result should be in terms of x and y .
5. 20pts) Let $f(x) = \frac{e^x}{1 - e^x}$, find the x intercept, y intercept, extreme points, increasing/decreasing intervals, concavity, inflection points and horizontal and vertical asymptote if exists. Then sketch the graph.
6. 10pts) Sand is flowing at 10 cubic centimeters per hour out from the bottom of a conical hour glass whose point is at the bottom. The radius of the top of the cone is $\frac{1}{2}$ the height of the cone. Determine how fast the top surface of the sand in the hour glass is dropping when the top surface is 5 centimeters from the bottom.
7. a) 15pts. Evaluate the following integrals
 - i) $\int_{-1}^3 |x+1| dx$ ii) $\int \csc x dx$ iii) $\int t^2 \sqrt{t+1} dt$
 b) 12pts) Determine the area between the graph of $y^2 = x$ and $y = x - 1$.
 c) 9pts) Find the derivative of $f(x) = \frac{(x^2 + 3)^{1/2} (\tan x + \ln x)^{10}}{(x^3 + \sin x)(\sqrt{x+1} + 5)^6}$
8. Bonus 10pts)

1. a) 1 b) 0 c) D.N.E
2. a) discontinuous b) continuous
3. a) $f(0) > 0$ and $f(-3) < 0$. By IVT, there is a zero between $[-3, 0]$.
b) $c_1 = -2$.

$$4. \quad y' = \frac{e^x - \sin y}{1 + x \cos y}$$

$$y'' = \frac{(e^x - \cos y \left(\frac{e^x - \sin y}{1 + x \cos y} \right))(1 + x \cos y) - (e^x - \sin y) \left(\cos y - x \sin y \left(\frac{e^x - \sin y}{1 + x \cos y} \right) \right)}{(1 + x \cos y)^2}$$

$$6. \quad \frac{-8}{5\pi} \quad . \quad 7. \text{ a) (i) } 4 \quad \text{(ii) } -\ln|\cot x + \csc x| + C$$

$$\text{(iii) } \frac{2}{7}(t+1)^{\frac{7}{2}} - \frac{4}{5}(t+1)^{\frac{5}{2}} + \frac{2}{3}(t+1)^{\frac{3}{2}} + C$$

$$\text{b) } \frac{(1+\sqrt{5})^2}{8} + \frac{1+\sqrt{5}}{2} - \frac{(1+\sqrt{5})^3}{24} - \left(\frac{(1-\sqrt{5})^2}{8} + \frac{1-\sqrt{5}}{2} - \frac{(1-\sqrt{5})^3}{24} \right)$$

$$\text{c) } f'(x) = f(x) \frac{d}{dx} (\ln |f(x)|)$$

$$i = \frac{(x^2+3)^{1/2} (\tan x + \ln x)^{10}}{(x^3 + \sin x)(\sqrt{x+1}+5)^6} \left(\frac{x}{x^2+3} + \frac{10(\sec^2 x + 1/x)}{\tan x + \ln x} - \frac{3x^2 + \cos x}{x^3 + \sin x} - \frac{3}{x+1+5\sqrt{x+1}} \right)$$