Exam 3

1.(30pts) Let $f(x) = \frac{1+e^x}{1-e^x}$

(a) Find the domain of f, and identify any intercepts of f.
(b) Find each vertical asymptote and horizontal asymptote of f that exists, identifying each.
(c) Identify the intervals on which f is increasing, and identify all relative extreme value of f, if any.
(d) Identify all intervals on which the graph of f is concave upward(if any).
(e) Identify all inflection points (if any).
(f) Sketch a graph of f, indicating all pertinent information on the graph.

2. Of all the rectangles in the first quadrant with one vertex at the origin, one vertex on the graph of $y = e^{-2x}$, and two sides along the axes, determine the maximum possible area A of such a rectangle. Justify the fact that your answer yields the maximum possible area.

3. Compute the $L_f(P)$, $U_f(P)$, the left sum and the right sum of the function $f(x) = sinx$ and $P = \{0, \pi/6, \pi/4, \pi/3, \pi/2\}$.

4. Calculate $\int_a^b (x^2 + 4)dx$.

5. Let f be a continuous function on $(-\infty, \infty)$. Use the Addition Property to find the values of a and b that make the equation true.
$\int_0^1 f(x)dx + \int_3^0 f(x)dx = \int_a^b f(x)dx$
$\int_3^4 f(x)dx + \int_b^a f(x)dx = \int_2^4 f(x)dx$

6. Determine the limit at infinity.
a) $\lim_{x\to\infty} e^{-x}$ b) $\lim_{x\to\infty} xsin1x$

Solution

1. (a)The denominator can not be zero: $1 - e^x \neq 0 \Rightarrow x \neq 0$. So the domain is $(-\infty, 0) \cup (0, \infty)$.
(b) Vertical asymptote: $x = 0$. Since $lim_{x\to\infty} \frac{1+e^x}{1-e^x} = \frac{e^{-x}+1}{e^{-x}-1} = -1$ and $lim_{x\to-\infty} \frac{1+e^x}{1-e^x} = 1$, then the horizontal asymptote is $y = -1$ and $y = 1$.
(c) $f'(x) = \frac{2e^x}{(1-e^x)^2} > 0$. $f(x)$ is increasing on the real line.
(d) $f''(x) = \frac{2e^x(e^x+1)}{(1-e^x)^3}$. $f''(x) > 0$ if $x < 0$ and $f''(x) < 0$ if $x > 0$. So the function is concave upward on $(-\infty, 0)$.
(e) Inflection point $x = 0$.

2. Suppose the coordinate of the upper right corner of such a rectangle is (x, y). Since it is on the graph of the given function, the coordinate should obey $y = e^{-2x}$. We want to maximize the area $A = xy = xe^{-2x}$. The maximization problem is $max_{x>0} xe^{-2x}$. First we neet to find the critical points: $A' = e^{-2x} - 2xe^{-2x} = (1-2x)e^{-2x}$. $A' = 0$ if $1 - 2x = 0 \Rightarrow x = \frac{1}{2}$. Since $A'' = -2e^{-2x} - 2e^{-2x} + 4xe^{-2x} = -4(1-x)e^{-2x}$, then $A''(\frac{1}{2}) = -2e^{-1} < 0$. The second derivative test implies its a maximum.

3. $L_f(P) = sin0(\frac{\pi}{6} - 0) + sin(\frac{\pi}{6})(\frac{\pi}{4} - \frac{\pi}{6}) + sin(\frac{\pi}{4})(\frac{\pi}{3} - \frac{\pi}{4}) + sin(\frac{\pi}{3})(\frac{\pi}{2} - \frac{\pi}{3}) = \frac{\pi}{24} + \frac{\sqrt{2}\pi}{24} + \frac{\sqrt{3}\pi}{12}$.
Similarly: $U_f(P) = \pi(\frac{1}{4} + \frac{\sqrt{2}}{24} + \frac{\sqrt{3}}{24})$. Left sum=$L_f(P)$. Right sum=$U_f(P)$.

4. $\frac{1}{3}(b^3 - a^3) + 4(b - a)$

5. $a = 3$, $b = 1$.
$a = 3$, $b = 2$.

6. a) 0.
b) Let $y = 1/x$, then $lim_{x\to\infty} x sin\frac{1}{x} = lim_{y\to 0} \frac{sin y}{y} = 1$.