Exam 2

1. 10pts)Use the linear approximation to approximate $\sqrt{99.5}$ and find the error bound.

2. 20pts) Solve for dy/dx. Please simplify your final answer.
a) $y = x^2\sqrt{x^2+2}$ b) $y^2 = tan^2(x)$
c) $sin(y) = \pi^{x^2+1}$ d) $y = x^2 secy$

3. 10pts)You want to approximate $\sqrt[4]{125}$ using the Newton-Raphson method.
a) Write down an appropriate function so that you can apply the Newton-Raphson method.
b) Let $c_0 = 3$, and find c_1.

4. 20pts)For the equation $x^2y^3 = 1$,
a) Find $\frac{d^2y}{dx^2}$.
b) Find the equation of the tangent line at the point $x = 1$

5 10pts)Water leaks onto a floor forming a pool in the shape of a cylinder of height 1mm with volume increasing at a rate of 4 cm^2/min. How fast is the radius of the pool increasing when the radius is 12cm.

6. 10pts)A manufacturer wishes to produce rectangular containers with square bottoms and tops, each container having a capacity of 150 cubic inches, If the material used for the top and the bottom costs twice as much per square inch as the material for the sides, what dimensions will minimize the cost?

7.10pts) Assume that the air pressure $p(x)$ in pounds per square foot at x feet above sea level is given by $p(x) \approx 2140e^{-0.000035x}$ for $x \geq 0$ and that an airplane is losing altitude at the rate of 20 miles per hour. At what rate is the air pressure just outside the plane increasing when the plane is 2 miles above sea level?

8.10pts)Determine the function f which satisfying the given condition $f''(x) = 0$ $f(1) = 1$, $f(0) = 2$.

9. Bonus(5pts) Show that $cosx > 1 - x^2/2$ for all $x > 0$.

Solution to the sample exam 2

1. Let $f(x) = \sqrt{x}$, $x_0 = 99.5$, $a = 100$. Then $\sqrt{99.5} = f(x_0) \approx f(a) + f'(a)(x_0 - a) = 10 + \frac{-0.5}{20} = 9.975$.
Sine $f'(x)$ is decreasing, for $x \in [99.5, 100]$, $|f'(x)| \leq |f'(99.5)| = \frac{1}{2\sqrt{99.5}}$. So error$\leq \frac{1}{2} \cdot \frac{1}{2\sqrt{99.5}} \cdot 0.25$.(You don't need to simplify the answer).

2. a) $y' = \frac{3x^3+4x}{\sqrt{x^2+2}}$
b) $y' = \frac{tanxsec^2x}{y}$
c) $y' = \frac{2x\pi^{x^2+1}ln(\pi)}{cosy}$.
d) $y' = \frac{2xsecy}{1-x^2secytany}$.
3. a) $f(x) = x^4 - 125$.
b) Since $f'(x) = 4x^3$, $c_1 = c_0 - \frac{f(c_0)}{f'(c_0)} = 3 - \frac{3^4-125}{4 \cdot 3^3} = 3 + 11/27$.

4. a).$y' = -\frac{2y}{3x}.y'' = 10y/9x^2$.
b).When $x = 1$, insert into the given equation we get $y = 1$. So $y'(1) = -2/3$. The equation of tangent line is $y - 1 = -\frac{2}{3}(x - 1)$.

5. Let V be the volume of the cylinder, $h = 1mm = 0.1cm$ is the height, r is the radius. Then we have $V = \pi r^2 h$. Differentiate both sides: $V' = \pi h \cdot 2rr'$. It is given that currently $V' = 4$, $r = 12$, $h = 0.1$. By calculation, $r' = \frac{5}{3\pi}$.

6. Let a be the side length of the square bottom and top. Let h be the height. Since the capacity must be 150, and $V = a^2h$, so $a^2h = 150 \Rightarrow h = 150/a^2$. We assume the unit price of the material for sides is 1, so for the top and bottom is 2. The area of the top and bottom is $2a^2$, and of the 4 sides are $4ah$, so the total cost is $C = 4a^2 + 4ah$. Substitute $h = 150/a^2$, we get $C = 4a^2 + 600/a$. The extreme value of the cost function occurs where $0 = C' = 8a - 600/a^2$, so $a = \sqrt[3]{75}$. The derivative changes from negative to positive at this point, so C has a minimum at this point.
7.1mile=5280 feet. 20 mile per hour=1760 feet/min. So $\frac{dx}{dt} = -1760$. It is given that $P(x) = 2140e^{-0.000035x}$, differentiate both sides: $\frac{dP}{dt} = 2140 \cdot (-0.000035)e^{-0.000035x} \cdot \frac{dx}{dt} = 1760 \cdot 2140 \cdot (0.000035)e^{-0.000035 \cdot 2 \cdot 5280}$

8. $f(x) = -x + 2$.

9. Let $f(x) = cosx - 1 + x^2/2$. Observe $f(0) = 0$. Examine $f'(x) = -sinx + x = x - sinx$, we proved in class that $sinx < x$ if $x > 0$. So $f'(x) > 0$. Combine with $f(0) = 0$, we obtain $f(x) > 0$ for $x > 0$. This is equivalent to $cosx > 1 - x^2/2$.