Pre-calculus Exam

1. (4pts)In the following exercises write h as the composite of $f \circ g$ of two function f and g (neither of which is equal to h).

   a) $h(x)=\sqrt{(x-3)}$ b) $h(x)=(x+\frac{1}{x})^{5/2}$

2. (4pts)Find the following values.

   a) sin(11π/6) b) tan(-π/4)

   c) sec (-π/3) d) csc(-5π/3)

3. (6pts)Solve the inequality.

   a) $2\ln x \geq \ln(2x)$ b) $\left|\frac{x+1}{x-1}\right| \leq 1$

4. (6pts)Find the domain of the function.

   a) $f(x)=\ln\frac{e^x}{e^x-1}$ b) $f(x)=\frac{\ln(1-x)}{\ln(x)}$

5. (5pts)Let r be any rational number and let f(x)=sin x+sin rx. Show that f is periodic.(Hint: let r=m/n, where m and n are integers).

Solution to Pre-Calculus Exam

1. a) g(x)=x-3, f(x)= $\sqrt{(x)}$
   b) g(x)=x+1/x, f(x)= $x^{5/2}$
2. a) sin(11π/6)=sin(2π-π/6)=sin(-π/6)=-sin(π/6)=-1/2
   b) tan(-π/4)=-tan(π/4)=-1.
   c) sec(-π/3)=1/cos(-π/3)=1/cos(π/3)=2.
   d) csc(-5π/3)=1/sin(-5π/3)=1/sin(-5π/3+2π)=1/sin(π/3)= $2/\sqrt{(3)}$
3. a) $2\,lnx \geq ln2x \;\Rightarrow\; 2\,lnx \geq ln2 + lnx \;\Rightarrow\; lnx \geq ln2 \;\Rightarrow\; x \geq 2$
   b) $\left|\frac{x+1}{x-1}\right| \leq 1 \;\Rightarrow\; -1 \leq \frac{x+1}{x-1} \leq 1$ multiplying each term by x-1,
   If x-1>0, then we get $-(x-1) \leq x+1 \leq x-1$, the right inequality doesn't make sense. So there's no solution under this case.
   If x-1<0, then we get $-(x-1) \geq x+1 \geq x-1$, the right inequality always holds. The left inequality gives $-(x-1) \geq x+1 \;\Rightarrow\; -x+1 \geq x+1 \;\Rightarrow\; -x \geq x \;\Rightarrow\; 0 \geq x \;\Rightarrow\; x \in (-\infty, 0]$
4. a) The domain of a function is all the value x where the function is well defined. The denominator can not be zero. So $e^x - 1 \neq 0 \;\Rightarrow\; e^x \neq 1 \;\Rightarrow\; x \neq 0$. The input of the ln function should be positive. So we must have $\frac{e^x}{e^x - 1} > 0$, but $e^x > 0$ for all x, the denominator should also be positive: $e^x - 1 > 0 \;\Rightarrow\; e^x > 1 \;\Rightarrow\; e^x > e^0$ ⇒x>0.
   b) The input of the ln function is positive. So we have 1-x>0, and x>0. Combine these two we get 0<x<1. The denominator can not be zero, so $lnx \neq 0 \;\Rightarrow\; x \neq 1$. Therefore the domain is (0,1).
5. A function g(x) is periodic if there exist some T>0, such that g(x+kT)=g(x), holds for any integer k. Recall the fact that sin(x) is periodic: $\sin(x + 2k\pi) = \sin(x)$, for any integer k. Since r is a rational number, there exist m, n such that r=m/n. So

   $$\sin(r(x+2kn\pi)) = \sin(\frac{m}{n}(x+2kn\pi)) = \sin(\frac{m}{n}x + 2km\pi) = \sin(\frac{m}{n}x) = \sin(rx) \quad (*)$$

   Using this result, we have

   $$\begin{aligned} f(x+2kn\pi) &= \sin(x+2kn\pi) + \sin(r(x+2kn\pi)) \\ &= \sin(x) + \sin(rx) \quad \text{(here we use * and the periodicity of sin function)} \\ &= f(x) \end{aligned}$$

   Therefore f(x) is a periodic function.