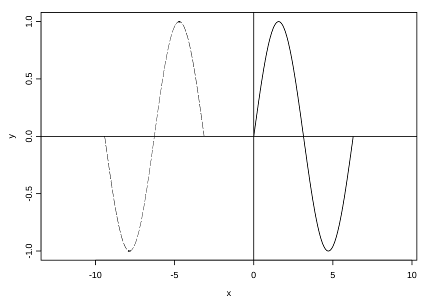
1. (a) [4 pts] Give 3 different values for θ in radian system such that $\cos(\theta) = -\frac{\sqrt{2}}{2}$.

solution: any 3 values from $\frac{3\pi}{4} + 2\pi k$, $\frac{5\pi}{4} + 2\pi k$, for $k = 0, \pm 1, \pm 2, \cdots$.

(b) [7 pts] The curve of the dash line below is obtained by using combinations of shifting and reflecting on the curve of the solid line, which is the graph of $y = \sin(x)$ for $0 \le x \le 2\pi$. Describe a sequence of such geometric maneuvers by words. Then write down the final function whose graph is the curve of the dash line. You don't need to specify the domain and the range.



solution: There're many ways to move $y = \sin(x)$. I list here 3 possible ways:

(1) shift $y = \sin(x)$ to the left 3π units, then reflect it with respect to the x-axis, we get $y = -\sin(x+3\pi)$.

(2) reflect $y = \sin(x)$ with respect to the x-axis, then shift it to the left 3π units, we get the same equation like in (1).

(3) reflect $y = \sin(x)$ with respect to the y-axis, then shift it to the left π units, we get $y = \sin(-(x+\pi))$.

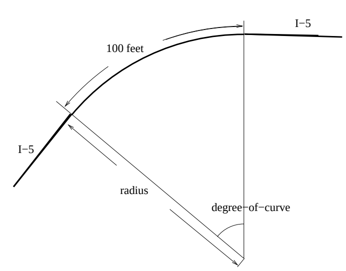
2. [13 pts] Show your steps for full credits.

(a) [7 pts] Given a one-to-one function $y = g(x) = -x^2 + 5$ for $x \ge 2$, find the inverse function $y = g^{-1}(x)$ with its domain and range.

solution: step 1, solve x from $y = -x^2 + 5$, we have $x = \pm\sqrt{5-y}$, as $x \ge 2$, take "+", so $x = \sqrt{5-y}$; step 2, exchange the positon of x and y, we have $y = g^{-1}(x) = \sqrt{5-x}$.

Its range is easy to get: $\{y|y \ge 2\}$. We need to have a graph to figure out the domain of the inverse funciton. The graph should show that the domain of the inverse is $\{x|x \le 1\}$.

(b) [6 pts] In the highway design, the "degree-of-curve" is the number of degrees created by a 100 feet long arc length. In order to provide safe travel, for a special part of I-5, engineers decide to build a 100 feet long arc with the radius of 5000 feet. Find the degree-of-curve for this part of I-5. Please approximate your answer with 2 decimals.



solution: There're 2 ways to do this problem. One by finding the degree-of-curve directly by the arc length formula of an angle measured in degree system; another by finding the angle measured in radian system first and then convert it into a degree. I'll just write down the first one:

Denote the degree-of-curve here as $x°$, since we know $S = \frac{x°}{360°}2\pi r$, we have $100 = \frac{x°}{360°}2\pi \cdot 5000$, then $x = \frac{100 \cdot 360}{2\pi \cdot 5000} \approx 1.15°$.

3. [14 pts] Given a rational function $y = f(x) = \frac{2x+3}{x-5}$,

(a) [1 pts] Find the natural domain of $y = f(x)$.

solution: domain=$\{x|x \neq 5\}$.

(b) [5 pts] Find the x-intercept and y-intercept of $y = f(x)$.

solution: x-intercept: the same as finding zeros, we have $x = \frac{-3}{2}$. y-intercept: just let $x = 0$, we have $y = \frac{3}{-5}$.

(c) [5 pts] Assume this rational function is one-to-one, find the inverse function $y = f^{-1}(x)$.

solution:

$$\begin{aligned} y &= \frac{2x+3}{x-5} \\ y(x-5) &= 2x+3 \\ yx - 5y &= 2x+3 \\ yx - 2x &= 3+5y \\ (y-2)x &= 3+5y \\ x &= \frac{5y+3}{y-2} \end{aligned}$$

so $y = f^{-1}(x) = \frac{5x+3}{x-2}$.

(d) [3 pts] Find the horizontal asymptotic line(s) of $y = f^{-1}(x)$. Please write your line equation in a proper form.

solution: he horizontal asymptotic line of $y = f^{-1}(x) = \frac{5x+3}{x-2}$: $y = 5$. (note: it's the mirrored image of the vertical asymptotic line of $y = f(x) = \frac{2x+3}{x-5}$)

(e) *Bonus[3 pts]*: Can you write down the range of $y = f(x)$ without knowing the graph of $y = f(x)$?

solution: The natural domain of $y = f^{-1}(x) = \frac{5x+3}{x-2}$ is $\{x|x \neq 2\}$, therefore the range of $y = f(x)$ is $\{y|y \neq 2\}$. (note: $y = 2$ is just the horizontal asymptotic line of $y = f(x) = \frac{2x+3}{x-5}$)

4. [12 pts] For this problem, we're concerned with the linear speed for different spots on the earth. Assume the earth is a perfect sphere, it is known that the earth rotates at a linear speed of 1700 kilometers per hour at the Equator.

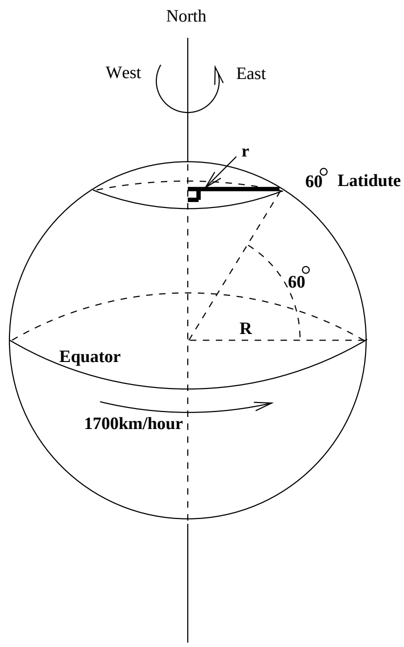
(a) [3 pts] Use your common sense to find the angular speed of spots on the earth at the Equator. Your answer should be in terms of radian per hour. (You don't need to know anything about the radius of the earth)

solution: the angular speed: $\frac{2\pi}{\text{one day}} = \frac{2\pi}{\text{24 hours}} = \frac{\pi}{12}$ rad per hour. (note: it doesn't matter where the spot is, everywhere on the earth, the angular speed is the same and this will be the key for (c).)

(b) [3 pts] Denote the radius of the earth as R, we know that all spots on the earth at 60° Latitude form a small circle with radius r. The picture below shows the meaning of Latitude. Express r in terms of R.

solution: If you can find r on the picture and relate it with R in a triangle, it's easy to get $r = R\sin 30° = \frac{R}{2}$. (I add r in the graph the next page.)

(c) [6 pts] Find the linear speed of spots on the earth at 60° Latitude. Your answer should be a number with the appropriate unit and should not contain R.



Using the formula $v = r\omega$, we have $v_r = r\omega_r = \dfrac{R}{2}\omega_R = \dfrac{R}{2} \cdot \dfrac{v_R}{R} = \dfrac{v_R}{2} = \dfrac{1700}{2} = 850$ kilometers per hour.