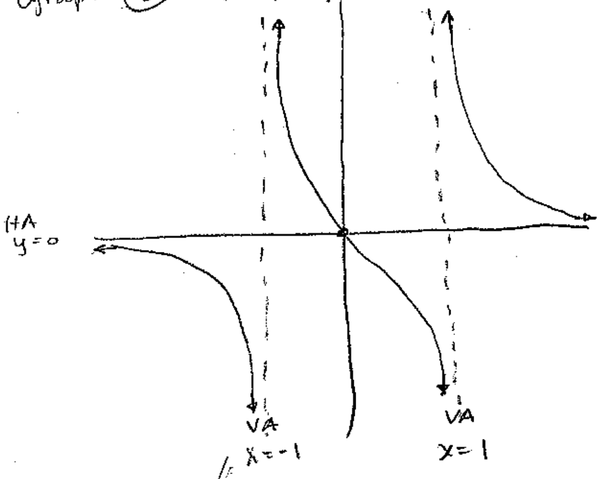
graph ⑧ $y = \frac{x}{x^2-1}$



graph ⑨ $y = \frac{2x^3}{x^2+1}$

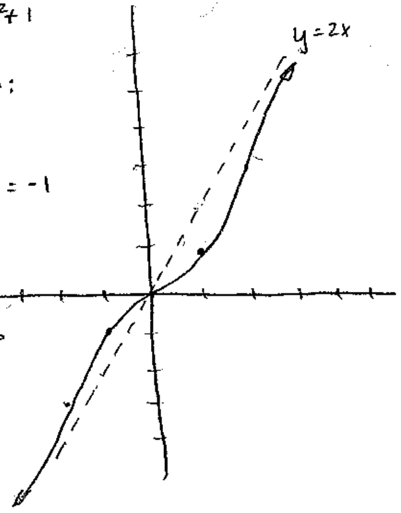
plot some points:

x = 1 $y = \frac{2(1)^3}{1^2+1} = 1$

x = -1 $y = \frac{2(-1)^3}{1^2+1} = \frac{-2}{2} = -1$

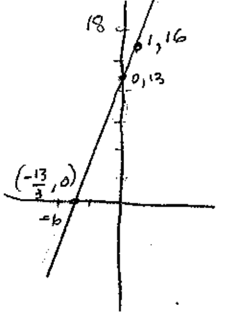
x = 2 $y = \frac{2(2)^3}{2^2+1} = \frac{16}{5}$

x = -2 $y = \frac{2(-2)^3}{2^2+1} = \frac{-16}{5}$

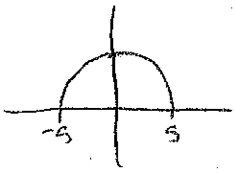


29 April

PreCalculus – Final Review



1. Find the slope and y-intercept (if possible) of the equation of the line. Sketch the line:
   $y = 3x + 13$ $y = mx + b$ m = slope b = y int.
   m = 3 = slope b = 13 y int = (0,13)
   $y = 0 = 3x + 13$ $3x = -13$ $x = -\frac{13}{3}$



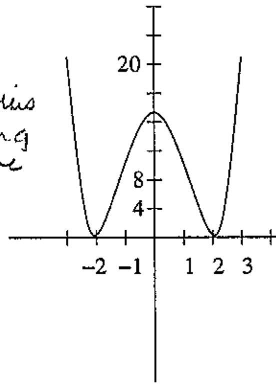
2. Determine the domain of the function. Verify your result with a graph.
   $f(x) = \sqrt{25 - x^2}$ no ~~negative~~ negative even root
   $25 - x^2 \geq 0$ $(5-x)(5+x) \geq 0$
   so $-5 \leq x \leq 5$ or $[-5, 5]$

5-x	pos	pos	neg
5+x	neg	pos	pos
	N -5	P 5	N

$y = \sqrt{25 - x^2}$
$y^2 = 25 - x^2$
$x^2 + y^2 = 25$
upper half circle

3. Determine the intervals over which the function is increasing, decreasing or constant.

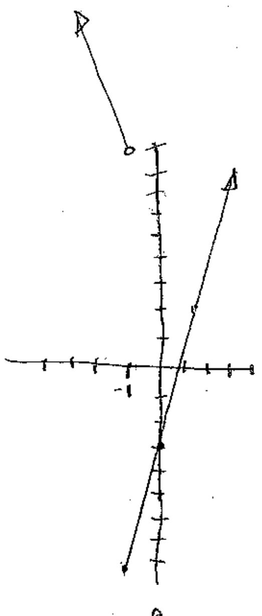
This question is asking how are the y-values of this function behaving as x-values move right.



as x moves to -2, y-values falling
from -2 to 0, y values rising
from 0 to 2, y values falling
from 2 on, y-values rising

increasing: $-2 < x < 0$ and $x > 2$

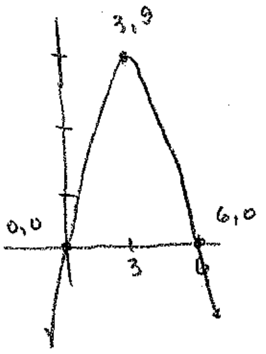
decreasing $x < -2$
$0 < x < 2$



4. Graph the function: $f(x) = \begin{cases} 5x - 3, & x \geq -1 \\ -4x + 5, & x < -1 \end{cases}$

$y = 4 - \frac{1}{3}x$ $x = 4 - \frac{1}{3}y$ $(x - 4 = -\frac{1}{3}y) * (-3)$
$-3x + 12 = y$ $f^{-1}(x) = -3x + 12$

5. Find the inverse. Verify: $f(x) = 4 - \frac{1}{3}x$
   $f(f^{-1}(x)) = f(-3x+12) = 4 - \frac{1}{3}(-3x+12) = 4 + \frac{3}{3}x - \frac{12}{3} = 4 + x - 4 = x$
   $f^{-1}(f(x)) = f^{-1}(4 - \frac{1}{3}x) = -3(4 - \frac{1}{3}x) + 12 = -12 + \frac{3}{3}x + 12 = x$



6. Write the quadratic function in standard form and sketch its graph. Identify the vertex and x-intercepts. $y = a(x-h)^2 + k$ complete square
   $f(x) = 6x - x^2$
   $f(x) = -x^2 + 6x = -(x^2 - 6x + \underline{9}) = (-1)\underline{9} = -(x^2 - 6x + 9) + 9 = -(x-3)^2 + 9$
   parabola opens DOWN because a = -1 vertex at (3,9)
   x intercepts: $0 = 6x - x^2 = x(6 - x)$ x = 0 x = 6

7. Find the domain of the rational function $f(x) = \frac{8}{x^2 - 10x + 24}$ no division by zero.
   Check when denominator = 0
   $x^2 - 10x + 24 = 0$ $(x-6)(x-4) = 0$
   x = 6 x = 4
   ℝ except {4,6} or $x \neq 4$ $x \neq 6$

8. Identify intercepts, check for symmetry, identify any vertical or horizontal asymptotes, and sketch the graph of the rational function.
   $y = \frac{x}{x^2 - 1}$ x-int means y = 0 $0 = \frac{x}{x^2-1}$ fraction zero only when numerator zero, so x = 0 y-int (0,0)
   vertical asymptotes where denominator = 0 $(x^2 - 1) = 0$ $(x-1)(x+1) = 0$ x = -1 x = 1
   deg num < deg den so y = 0 horizontal asymptote

factor picture:

x	-	-	+	+
x-1	-	-	-	+
x+1	-	+	+	+
	N -1	P 0	N 1	P

FOR GRAPH, SEE PAGE TOP

9. State the domain of the function and identify any vertical and slant asymptotes. Then sketch the graph of the rational function.
   $f(x) = \frac{2x^3}{x^2 + 1}$ $0 = \frac{2x^3}{x^2+1}$ x = 0 y = 0 $x^2 + 1 \neq 0$ so domain ℝ deg num > deg den so slant asymptote. Find by long division

$$\begin{array}{r} 2x \\ x^2+1 \overline{)\,2x^3 + 0x^2 + 0x + 0} \\ \underline{2x^3 \quad\quad + 2x} \\ -2x \end{array}$$

$f(x) = 2x - \frac{2x}{x^2 - 1}$ so slant asymptote $y = 2x$

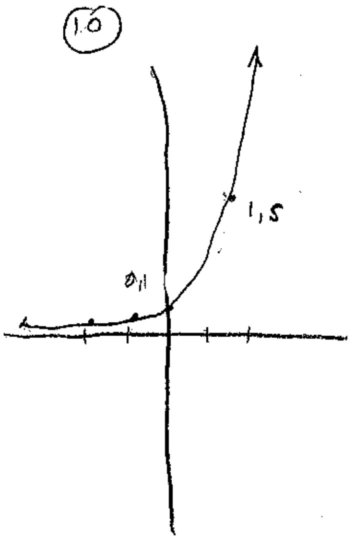
factor picture

$2x^3$	-	+
x^2+1	+	+
	N 0	P

always positive

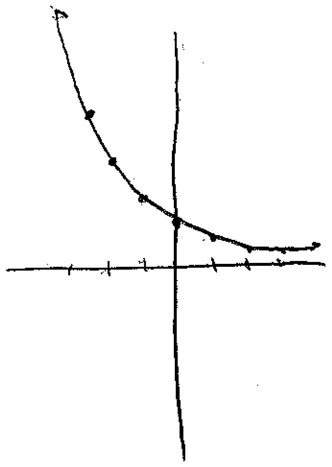
(10)

x	5^x
-2	$\frac{1}{5^2}=\frac{1}{25}$
-1	$\frac{1}{5^1}=\frac{1}{5}$
0	$5^0=1$
1	$5^1=5$
2	$5^2=25$
3	$5^3=125$



(11)

x	$\left(\frac{2}{3}\right)^x$
-3	$\left(\frac{3}{2}\right)^3=\frac{27}{8}$
-2	$\left(\frac{3}{2}\right)^2=\frac{9}{4}$
-1	$\left(\frac{3}{2}\right)^1=\frac{3}{2}$
0	$\left(\frac{2}{3}\right)^0=1$
1	$\left(\frac{2}{3}\right)^1=\frac{2}{3}$
2	$\left(\frac{2}{3}\right)^2=\frac{4}{9}$
3	$\left(\frac{2}{3}\right)^3=\frac{8}{27}$



10. Graph: $f(x) = 5^x$

11. Graph: $f(x) = \left(\frac{2}{3}\right)^x$

Compound Interest: Complete the table to determine the balance A for P dollars invested at rate r for t years and compounded n times per year.

$A=P(1+\frac{r}{n})^{nt}$ compounded n times a year for t years

$A=Pe^{rt}$ compounded continuously for t years

n	1	2	3	4	365	Continuous
A						

n=1

$A=3500(1+.065)^{10}$

$=3500(1.065)^{10}$

$\approx 3500(1.877)$

\$6,569.98

n=2 $A=3500(1+\frac{.065}{2})^{20}=3500(1.0325)^{20} \doteq 3500(1.896)=$ \$6,635.42

n=3 $A=3500(1+\frac{.065}{3})^{30}=3500(1.0216)^{30} \doteq 3500(1.899)=$ \$6,644.99

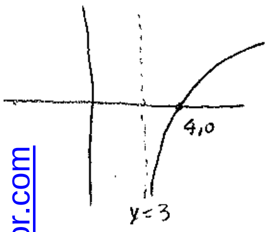
12. $P = \$3500$, $r = 6.5\%$, $t = 10$ years

n=4 $A=3500(1+\frac{.065}{4})^{40}=3500(1.01625)^{40} \doteq 3500(1.91)=$ \$6,669.46

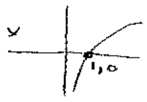
n=365 $A=3500(1+\frac{.065}{365})^{3650} \doteq 3500(1.000178)^{3650} \doteq 3500(1.91543)=$ \$6,704.00

$A=3500e^{.065(10)}=3500e^{.65} \doteq 3500(1.91554)=$ \$6704.39

13. Find the domain, x-intercept, and vertical asymptote of the logarithmic function and sketch its graph.
$f(x) = \ln(x-3)$



y=ln x



Domain ln x is x>0

14. Condense the expression to the logarithm of a single quantity.
$5\ln(x-2) - \ln(x+2) - 3\ln x = \ln(x-2)^5 - \ln(x+2) - \ln x^3$

$= \ln(x-2)^5 - [\ln(x+2) + \ln x^3] = \ln(x-2)^5 - \ln(x+2)x^3$

$= \ln\left(\frac{(x-2)^5}{(x+2)x^3}\right)$

15. Solve for x: $3^x = 729$ $729 = 9 \cdot 81 = 9 \cdot 9 \cdot 9 = 3^2 \cdot 3^2 \cdot 3^2 = 3^6$

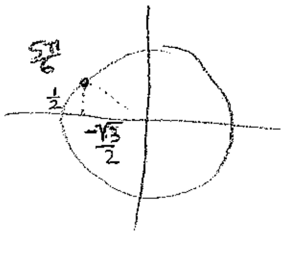
get same base, then exponents equal

$3^x = 3^6$ $x=6$

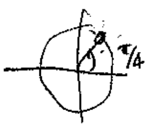
16. Solve the logarithmic equation. Approximate the result to three decimal places.
$\ln x - \ln 3 = 2$ $\ln\left(\frac{x}{3}\right)=2$ $e^{\ln(\frac{x}{3})}=e^2$ $\frac{x}{3}=e^2$

$x=3e^2 \doteq 22.167$

17. Find the point (x, y) on the unit circle that corresponds to the real number t.
$t = \frac{5\pi}{6}$ $(x,y)=(\cos\frac{5\pi}{6}, \sin\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}, \frac{1}{2}$



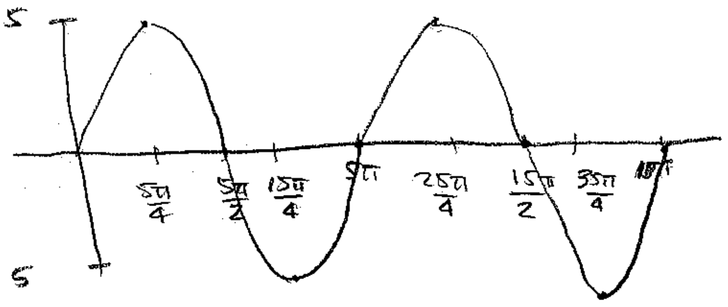
18. Evaluate (if possible) the six trigonometric functions of the real number.
$t = \frac{\pi}{4}$



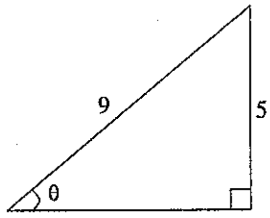
$\sin t = \frac{1}{\sqrt{2}}$ $\tan t = 1$ $\sec t = \sqrt{2}$

$\cos t = \frac{1}{\sqrt{2}}$ $\cot t = 1$ $\csc t = \sqrt{2}$

graph ㉒



19. Find the exact values of the six trigonometric functions of the angle θ shown in the figure.



$\sqrt{9^2-5^2}$

$\sqrt{81-25}=\sqrt{56}$

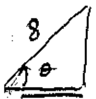
$=2\sqrt{14}$

$sen\ \theta=\frac{5}{9}$ $\cos\theta=\frac{2\sqrt{14}}{9}$ $\tan\theta=\frac{5}{2\sqrt{14}}$

$\cot\theta=\frac{2\sqrt{14}}{5}$ $\sec\theta=\frac{9}{2\sqrt{14}}$ $\csc\theta=\frac{9}{5}$

20. Find the remaining five trigonometric functions of θ satisfying the condition.

$\sin\theta=\frac{3}{8}$



$\sqrt{64-9}=\sqrt{55}$

$\cos\theta=\frac{\sqrt{55}}{8}$ $\tan\theta=\frac{3}{\sqrt{55}}$ $\cot\theta=\frac{\sqrt{65}}{3}$ $\sec\theta=\frac{8}{\sqrt{55}}$ $\csc\theta=\frac{8}{3}$

21. Evaluate the sine, cosine, and tangent of the angle without using a calculator: $\frac{\pi}{3}$

$sen\ \frac{\pi}{3}=\frac{\sqrt{3}}{2}$ $\cos\frac{\pi}{3}=\frac{1}{2}$ $\tan\frac{\pi}{3}=\sqrt{3}$

22. Sketch a graph of the function. Include two full periods.

$f(x)=5\sin\frac{2x}{5}$ amplitude 5 "starting point" 0 new pd = $\frac{2\pi}{\frac{2}{5}}=5\pi$

23. Name the trigonometric function that is equivalent to the expression: $\sqrt{1+\tan^2 x}$

$\frac{sen^2+\cos^2=1}{\cos^2}\Rightarrow\tan^2+1=\sec^2$ $\sqrt{1+\tan^2x}=\sec x$

24. Use the fundamental trigonometric identities to simplify the trigonometric expression: $\tan^2\theta\csc^2\theta-\tan^2\theta$

$\tan^2\theta(\csc^2\theta-1)$

$\tan^2\theta(\cot^2\theta)=1$

$\frac{sen^2}{sn^2}+\frac{\cos^2}{sn^2}=\frac{1}{sin^2}$

$1+\cot^2=\csc^2$

$\cot^2=1-\cos^2$

25. Verify the identity: $\sec^2 x\cot x-\cot x=\tan x$

$(\sec^2x-1)\cot x\overset{?}{=}\tan x$

$\tan^2x\cot x\overset{?}{=}\tan x$

$\tan x\ \cancel{\tan x}\cot x\overset{?}{=}\tan x$

$\tan x(1)\overset{?}{=}\tan x$

$\tan x=\tan x$ ✓

26. Solve the equation: $4\cos\theta=1+2\cos\theta$

$4\cos\theta=1+2\cos\theta$

$-2\cos\theta\qquad -2\cos\theta$

$2\cos\theta=1$

$\cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3},\frac{5\pi}{3}$

27. Solve the system by the method of substitution: $\begin{cases}x=y+3\\x=y^2+1\end{cases}$

$y+3=x=y^2+1$

$y+3=y^2+1$

$0=y^2-y-2$

$0=(y-2)(y+1)$

$y=2\quad y=-1$

$x=2+3=5$

$x=-1+3=2$

$(5,2)$

$(2,-1)$

28. Solve the system by elimination: $\begin{cases}2x-y=2\\6x+8y=39\end{cases}$

$-6x+3y=-6$

$6x+8y=39$

$11y=33$

$y=3$

$2x-3=2$

$2x=5$

$x=\frac{5}{2}$

㉚

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -6 \\ 2 & -3 & 0 & -7 \\ -1 & 3 & -3 & 11 \end{array}\right]$$

$2r_1 - r_2 \to r_2$

$$\begin{array}{cccc} 2 & -4 & 2 & -12 \\ -2 & 3 & 0 & 7 \\ \hline 0 & -1 & 2 & -5 \end{array}$$

$r_1 + r_3 \to r_3$

$$\begin{array}{cccc} 1 & -2 & 1 & -6 \\ -1 & 3 & -3 & 11 \\ \hline 0 & 1 & -2 & 5 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -6 \\ 0 & -1 & 2 & -5 \\ 0 & 1 & -2 & 5 \end{array}\right]$$

$r_2 + r_3 \to r_3$

$$\begin{array}{cccc} 0 & -1 & 2 & 5 \\ 0 & 1 & -2 & 5 \\ \hline 0 & 0 & 0 & 0 \end{array}$$

$z = t$

infinite solutions

problem ㉚

$-y + 2z = -5$

$-y + 2t = -5$

$y - 2t = 5$

$y = 5 + 2t$

$x - 2(5+2t) + t = -6$

$x - 10 - 4t + t = -6$

$x - 10 - 3t = -6$

$x = 4 + 3t$

29. Use back-substitution to solve the system: $\begin{cases} x - 4y + 3z = 3 \\ -y + z = -1 \\ z = -5 \end{cases}$

$z = -5$

$-y + (-5) = -1$

$-y = 4$

$y = -4$

$x - 4(-4) + 3(-5) = 3$

$x + 16 - 15 = 3$

$x + 1 = 3$

$x = 2$

$(2, -4, -5)$

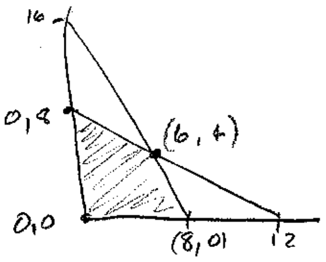
30. Use Gaussian elimination to solve the system of equations:

$$\begin{cases} x - 2y + z = -6 \\ 2x - 3y = -7 \\ -x + 3y - 3z = 11 \end{cases}$$

see top

31. Sketch a graph and label the vertices of the solution set of the system of inequalities:

$$\begin{cases} 2x + 3y \le 24 \\ 2x + y \le 16 \\ x \ge 0 \\ y \ge 0 \end{cases}$$



$2x + 3y = 24$

$-2x - y = -16$

$2y = 8$

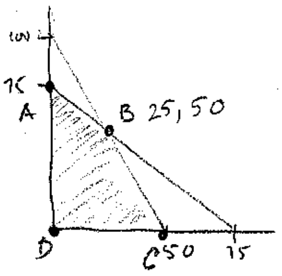
$y = 4$

$x = 6$

32. Sketch the region determined by the constraints. Then find the minimum and maximum values of the objective function and where they occur, subject to the indicated restraints.

Objective function: $z = 10x + 7y$

Constraints: $x \ge 0$, $y \ge 0$, $2x + y \ge 100$, $x + y \ge 75$



$2x + y = 100$

$-x - y = -75$

$x = 25$

Check vertices:

A $(0,75)$ $z = 10(0) + 7(75) = 525$

B $(25,50)$ $z = 10(25) + 7(50) = 600$

C $(50,0)$ $z = 10(50) + 7(0) = 500$

D $(0,0)$ $z = 10(0) + 7(0) = 0$

m 0 at $(0,0)$

M 600 at $(25,50)$

33. Use matrices to solve the system of equations. Use Gauss-Jordan elimination:

$$\begin{cases} -x + y + 2z = 1 \\ 2x + 3y + z = -2 \\ 5x + 4y + 2z = 4 \end{cases}$$

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{array}\right]$$

(see top of page 5)

34. If possible, find (a) $A + B$; (b) $A - B$; (c) $4A$; and (d) $A + 3B$.

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix}$$

$A + B = \begin{bmatrix} 2-3 & -2+10 \\ 3+12 & 5+8 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 15 & 13 \end{bmatrix}$

$3B = \begin{bmatrix} 3(-3) & 3(10) \\ 3(12) & 3(8) \end{bmatrix} = \begin{bmatrix} -9 & 30 \\ 36 & 24 \end{bmatrix}$

$A - B = \begin{bmatrix} 2-(-3) & -2-10 \\ 3-12 & 5-8 \end{bmatrix} = \begin{bmatrix} 5 & -12 \\ -9 & -3 \end{bmatrix}$

$4A = \begin{bmatrix} 4(2) & 4(-2) \\ 4(3) & 4(5) \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ 12 & 20 \end{bmatrix}$

$A + 3B = \begin{bmatrix} 2+(-9) & -2+30 \\ 3+36 & 5+24 \end{bmatrix} = \begin{bmatrix} -7 & 28 \\ 39 & 29 \end{bmatrix}$

35. Find AB, if possible

$$A = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 12 \\ 20 & 40 \end{bmatrix}$$

$\underbrace{3\times2 \quad 2\times2}$

ans 3×2

$AB = \begin{bmatrix} 5\cdot4 + 4\cdot20 & 5\cdot12 + 4\cdot40 \\ -7\cdot4 + 2\cdot20 & -7\cdot12 + 2\cdot40 \\ 11\cdot4 + 2\cdot20 & 11\cdot12 + 2\cdot40 \end{bmatrix} = \begin{bmatrix} 100 & 220 \\ 12 & -2 \\ 84 & 212 \end{bmatrix}$

Perform the matrix operations. If it is not possible, explain why.

36. $\begin{bmatrix} 1 & 5 & 6 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} 6 & -2 & 8 \\ 4 & 0 & 0 \end{bmatrix}$

$\underbrace{2\times3 \quad 2\times3}$

doesn't match

not possible, matrices are not conformable

(33)

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{array}\right] \quad \left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & 4 & 3 \end{array}\right] \quad \left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$2r_1 + r_2 \to r_2$

$$\begin{array}{cccc} -2 & 2 & 4 & 2 \\ 2 & 3 & 1 & -2 \\ \hline 0 & 5 & 5 & 0 \end{array} \Rightarrow 0\ 1\ 1\ 0$$

$3r_2 - r_3 \to r_3$

$$\begin{array}{cccc} 0 & 3 & 3 & 0 \\ 0 & -3 & -4 & -3 \\ \hline 0 & 0 & -1 & -3 \end{array}$$

$5r_1 + r_3 \to r_3$

$$\begin{array}{cccc} -5 & 5 & 10 & 5 \\ 5 & 4 & 2 & 4 \\ \hline 0 & 9 & 12 & 9 \end{array} \Rightarrow 0\ 3\ 4\ 3$$

$z = 3$

$y + 3 = 0$

$y = -3$

$-x + (-3) + 2(3) = 1$

$-x + 3 = 1$

$-x = -2$

$x = 2$

$(2, -3, 3)$

37. $[4 \quad -2 \quad 6]\begin{bmatrix} -2 & 1 \\ 0 & -3 \\ 2 & 0 \end{bmatrix}$ $= [4(-2) + -2(0) + 6(2) \quad 4(1) + -2(-3) + 6(0)] =$

$\underbrace{1\times 3 \quad 3\times 2}_{1\times 2 \text{ result}}$

$[-8 + 0 + 12 \quad 4 + 6 + 0] = [4 \quad 10]$

38. Show that B is the inverse of A.

$A = \begin{bmatrix} 5 & -1 \\ 11 & -2 \end{bmatrix}$, $\quad B = \begin{bmatrix} -2 & 1 \\ -11 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 5 & -1 \\ 11 & -2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -11 & 5 \end{bmatrix} = \begin{bmatrix} -10+11 & 5-5 \\ -22+22 & 11-10 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

39. Find the inverse of the matrix (if it exists).

$\begin{bmatrix} -3 & -5 \\ 2 & 3 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} -3 & -5 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right]$$

$2r_1 + 3r_2 \to r_2$

$$\begin{array}{cccc} -6 & -10 & 2 & 0 \\ 6 & 9 & 0 & 3 \\ \hline 0 & -1 & 2 & 3 \end{array}$$

$$\left[\begin{array}{cc|cc} -3 & -5 & 1 & 0 \\ 0 & -1 & 2 & 3 \end{array}\right]$$

$r_1 - 5r_2 \to r_1$

$$\begin{array}{cccc} -3 & -5 & 1 & 0 \\ 0 & 5 & -10 & -15 \\ \hline -3 & 0 & -9 & -15 \end{array}$$

$$\left[\begin{array}{cc|cc} -3 & 0 & -9 & -15 \\ 0 & -1 & 2 & 3 \end{array}\right] \quad \left[\begin{array}{cc|cc} 1 & 0 & 3 & 5 \\ 0 & 1 & -2 & -3 \end{array}\right]$$

$A^{-1} = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$

40. Use an inverse matrix to solve (if possible) the system of linear equations.

$$\begin{cases} -x + 4y = 8 \\ 2x - 7y = -5 \end{cases}$$

$\begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix}$ $\quad A^{-1} = \dfrac{1}{(-1)(-7) - 2(4)}\begin{bmatrix} -7 & -4 \\ -2 & -1 \end{bmatrix} = \dfrac{1}{-1}\begin{bmatrix} -7 & -4 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}$

$A^{-1}b = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ -5 \end{bmatrix} = \begin{bmatrix} 7(8) + 4(-5) \\ 2(8) + 1(-5) \end{bmatrix} = \begin{bmatrix} 36 \\ 11 \end{bmatrix}$ $\quad x = 36 \quad y = 11$

41. Use the Binomial Theorem to expand the binomial. Simplify your answer.

$(3x + y^2)^7$

$1(3x)^7 + 7(3x)^6(y^2) + 21(3x)^5(y^2)^2 + 35(3x)^4(y^2)^3 + 35(3x)^3(y^2)^4 + 21(3x)^2(y^2)^5 + 7(3x)(y^2)^6 + (y^2)^7$

$3^7x^7 + 7\cdot 3^6x^6y^2 + 21\cdot 3^5x^5y^4 + 35\cdot 3^4x^4y^6 + 35\cdot 3^3x^3y^8 + 21\cdot 3^2x^2y^{10} + 7\cdot 3xy^{12} + y^{14}$

42. Find the center, vertices, foci, and ~~eccentricity~~ of the ellipse.

$$\frac{(x+2)^2}{81} + \frac{(y-1)^2}{100} = 1$$

$9^2 \qquad 10^2$

$b^2 \qquad a^2$

center $(-2, 1)$

major (y) $(-2, 11)$ & $(-2, -9)$

minor (x) $(-11, 1)$ & $(7, 1)$

foci : $a^2 = b^2 + c^2$

$100 = 81 + c^2$

$c^2 = 9$

$c = \pm 3$

$(-2, 4)$ & $(-2, -2)$

43. Find the center, vertices, foci, and the equations of the asymptotes of the hyperbola, and sketch its graph.

$$\frac{(x-3)^2}{16} - \frac{(y+5)^2}{4} = 1.$$

$4^2 \qquad 2^2$

$a^2 \qquad b^2$

center $(3, -5)$ opens left/right

vertices: $(-1, -5)$ & $(7, -5)$

focus: $c^2 = a^2 + b^2$ $\quad c^2 = 16 + 4 = 20$ $\quad c = \pm 2\sqrt{5} \approx \pm 4.5$

$(3 - 2\sqrt{5}, -5)$ & $(3 + 2\sqrt{5}, -5)$ $\approx$ $(-1.5, -5)$ & $(7.5, -5)$

slope asymptotes:

$y = k \pm \frac{b}{a}(x - h)$

$y = -5 + \frac{1}{2}(x - 3)$

$y = -5 - \frac{1}{2}(x - 3)$

44. Classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola.

$-4y^2 + 5x + 3y + 7 = 0$

$A = 0 \quad C = -4$

∴ parabola

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