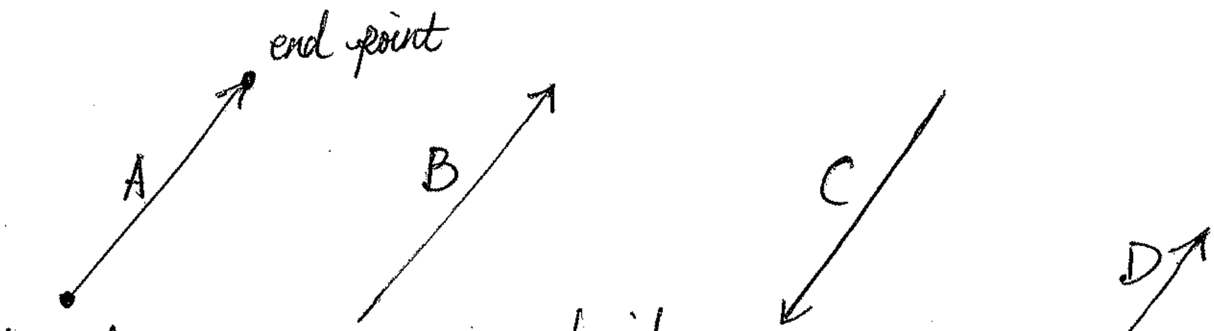
7.3 Vectors

Defn:

Scalars : quantities with magnitude only, such as length, area, volume, temperature & time.

Vectors : quantities with both magnitude and direction, such as velocity, acceleration and force.



$A = B$, only if they have the same magnitude and the same direction.

$A \neq C$, since they have different directions.

$A \neq D$, since they have different magnitudes.

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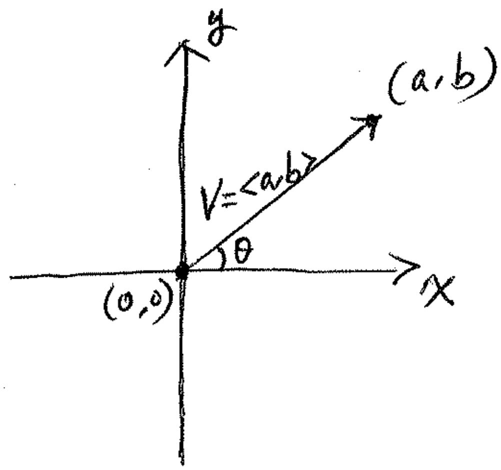
Position vector : A vector placed in a rectangular coordinate system so that its initial point is the origin.

$\langle a, b \rangle$: denotes a position vector with terminal point (a, b)

Direction angle, θ: The angle formed by the positive x-axis and a position vector. $0^\circ \leq \theta < 360^\circ$



The magnitude of $V = |V| = \sqrt{a^2+b^2}$

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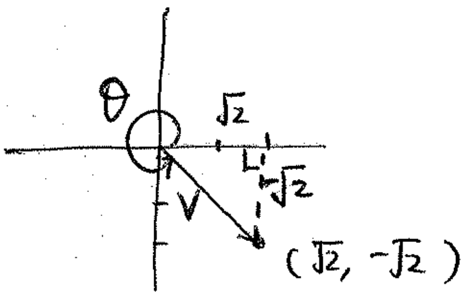
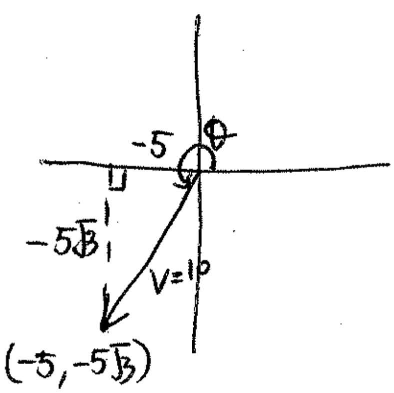
ex1: Find the magnitude and direction angle of the vector $V = < \sqrt{2}, -\sqrt{2} >$.



$$|V| = \sqrt{(\sqrt{2})^2+(-\sqrt{2})^2} = \sqrt{2+2} = 2.$$

$$\sin\theta = -\frac{\sqrt{2}}{2} \Rightarrow \theta = 315^\circ$$

ex 2: Find the magnitude and direction angle of the vector $V = <-5, -5\sqrt{3}>$



$$|V| = \sqrt{(-5)^2+(-5\sqrt{3})^2} = \sqrt{25+25\cdot 3}$$

$$= \sqrt{100} = 10$$

$$\sin\theta = -\frac{\cancel{5}\sqrt{3}}{\cancel{10}\,2} = -\frac{\sqrt{3}}{2} \Rightarrow \theta = 240^\circ$$

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Operations of Vectors:

If $A = \langle a_1, a_2 \rangle$, $B = \langle b_1, b_2 \rangle$, and k is a scalar:

1. $kA = \langle ka_1, ka_2 \rangle$ Scalar product
2. $A + B = \langle a_1 + b_1, a_2 + b_2 \rangle$ Vector Sum
3. $A - B = \langle a_1 - b_1, a_2 - b_2 \rangle$ Vector Difference
4. $A \cdot B = a_1 \cdot b_1 + a_2 \cdot b_2$ Dot product

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ex: Let $W = \langle -3, 2 \rangle$ and $Z = \langle 5, -1 \rangle$ Find

a) $W - Z$ b) $-8Z$, c) $3W + 4Z$ d) $W \cdot Z$

a) $W - Z = \langle -8, 3 \rangle$

b) $-8Z = \langle -40, 8 \rangle$

c) $3W + 4Z = \langle -9, 6 \rangle + \langle 20, -4 \rangle = \langle 11, 2 \rangle$

d) $W \cdot Z = -15 - 2 = -17$

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7.7 Parametric Equations

Definition:

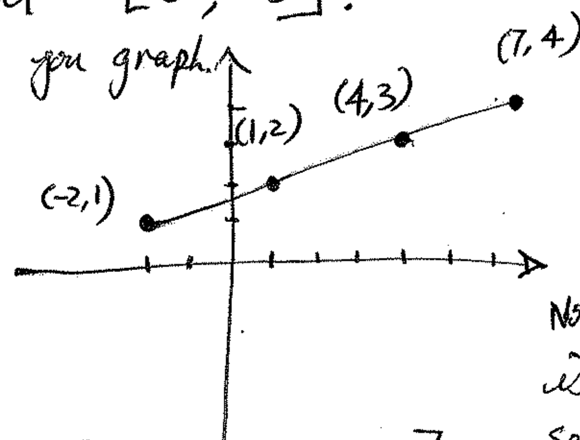
Parametric Equations: $x = f(t)$, $y = g(t)$, where t is in some interval of real numbers.

Ex: Graph the parametric equations: $x = 3t - 2$, $y = t + 1$ for t in the interval $[0, 3]$. Determine the domain and range of the function you graph.

$t = 0, \ 1, \ 2, \ 3$

$x = -2, \ 1, \ 4, \ 7$

$y = 1, \ 2, \ 3, \ 4$



Note: the graph is only a segment of a line

Domain: $[-2, 7]$; Range: $[1, 4]$

You can also find domain using:

$x = 3t - 2 \Rightarrow t = \dfrac{x+2}{3} \Rightarrow 0 \le \dfrac{x+2}{3} \le 3 \Rightarrow 0 \le x + 2 \le 9$

$t \in [0, 3]$

$-2 \le x \le 7.$

$y = t + 1 \Rightarrow t = y - 1 \Rightarrow 0 \le y - 1 \le 3 \Rightarrow 1 \le y \le 4.$

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Eliminate Parameters:

Ex: Eliminate Parameters and identify the graph of the Parametric equations. Determine the domain and range.

1) $x = 3t - 2$, $y = t + 1$, $-\infty < t < \infty$

$t = \dfrac{x+2}{3} \Rightarrow y = \dfrac{x+2}{3} + 1$

$y = \frac{1}{3}x + \frac{5}{3}$

$-\infty < t < \infty$ $\quad$ $-\infty < y-1 < \infty$

$-\infty < \frac{x+2}{3} < \infty$ $\quad$ $-\infty < y < \infty$

$-\infty < x < \infty$ $\quad$ Range $(-\infty, \infty)$.

Domain $(-\infty, \infty)$

the graph is the entire stright line given by the equation

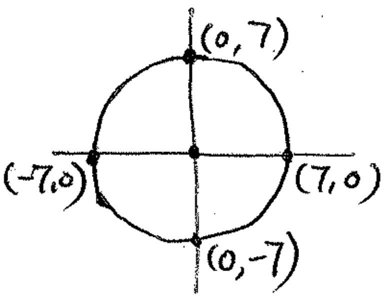
$$y = \frac{1}{3}x + \frac{5}{3}.$$

2) $x = 7\sin t$, $y = 7\cos t$, $-\infty < t < \infty$

$$\sin^2 t + \cos^2 t = 1$$

$$\left(\frac{x}{7}\right)^2 + \left(\frac{y}{7}\right)^2 = 1$$

$$x^2 + y^2 = 49$$



Domain : $[-7, 7]$; Range $[-7, 7]$

The graph is a circle centered at the origin with a radius 7.

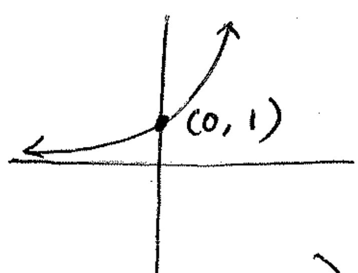
3) $x = \frac{t}{4}$, $y = e^t$

Note: when the interval of t is not specified, it's assumed ~~that~~ $-\infty < t < \infty$

$t = 4x$, $y = e^{4x}$

$-\infty < 4x < \infty$

$-\infty < x < \infty$



Domain $(-\infty, \infty)$, Range $(0, \infty)$

the graph is an exponential graph.

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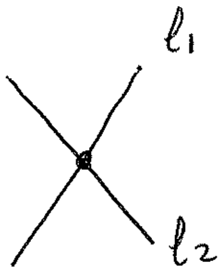
8.1 Systems of Linear Equations in Two Variable

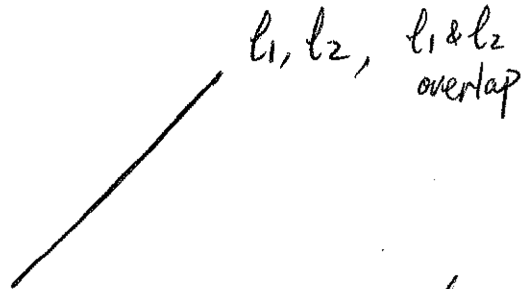
- The graph of a system of two linear equations consists two straight lines. The solution(s) of the system is/are the intersection(s) of the two lines.



one solution/one intersection

Consistent

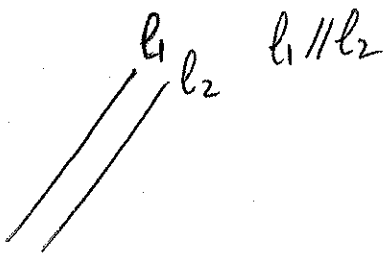
Independant



Infinity-many solutions/ Infinity-many intersections

Consistent

Dependent



No solution/ No intersection

Inconsistent

- Ex. Use substitution method to solve the following system of equations. Determine if the system is consistent or inconsistent, dependent or independent.

a) $\begin{cases} 2x - 3y = -2 & ① \\ 3x - 2y = 12 & ② \end{cases}$

step 1: choose an equation, say equation ①, express one variable in terms of the other variable.

$$2x - 3y = -2$$
$$2x = 3y - 2$$
$$x = \frac{3y-2}{2} \quad *$$

step 2: Substitute expression * into equation ②, and solve for the variable

$$3 \cdot \left(\frac{3y-2}{2}\right) - 2y = 12$$
$$3(3y-2) - 4y = 24$$
$$5y = 30$$
$$y = 6$$

step 3: Use $y=6$ in * ~~to~~ solve for x (the other variable).

$$x = \frac{3 \cdot 6 - 2}{2} = 8$$

The solution set is $\{(8, 6)\}$, The system is consistent and independent.

NOTE: The solution is an ordered pair, because it's a point on the xy-plane.

b). $\begin{cases} \frac{1}{2}x - \frac{2}{3}y = -2 & ① \\ -3x + 4y = 12 & ② \end{cases}$ $\xrightarrow[\text{multiply the L.C.D on bothside of the equation}]{\text{If you don't like fractions}}$ $\begin{cases} 3x - 4y = -12 \\ -3x + 4y = 12 \end{cases}$

from ①: $3x = 4y - 12$
$$x = \frac{4}{3}y - 4 \quad *$$

Sub. * in ②: $-3 \cdot \left(\frac{4}{3}y - 4\right) + 4y = 12$
$$\cancel{-4y} + 12 \cancel{+4y} = 12$$
$$12 = 12 \quad \text{always true}$$

Infinity many solutions! Consistent & Dependent

$$\{(x, y) \mid \tfrac{1}{2}x - \tfrac{2}{3}y = -2\}$$

c) $\begin{cases} 0.05x + 0.1y = 0.6 & (1) \\ 5x - 10 = -10y & (2) \end{cases}$

from (2) : $5x = -10y + 10$

$x = -2y + 2$ $\cancel{\ast}$

~~substitute~~ $\cancel{\ast}$ ~~into~~ (1) :

$0.05(-2y+2) + 0.1y = 0.6$

$-\cancel{0.1y} + 0.1 + \cancel{0.1y} = 0.6$

$0.1 = 0.6$ always false

No solution / Inconsistent

– Application (Word Problems)

Ex1. The executive inn rents a double room for \$10 more per night than a single room. One night the motel took in \$2159 by renting 15 double rooms and 26 singles. What is the rent price for each type of room?

Let x = rent price for a double room;
y = " " " " single "

$\begin{cases} x = y + 10 & (1) \\ 15x + 26y = 2159 & (2) \end{cases}$

replace (1) into (2) : $15(y+10) + 26y = 2159$

$15y + 150 + 26y = 2159$

$41y = 2009$

$y = 49 \Rightarrow x = 49 + 10$
$= 59$

The motel rented 59 double rooms, and 49 single rooms

Ex 2: James has a collection of 166 old coins consisting quarters & dimes. If he figures that each coin is worth two and a half times its face value, then his collection is worth \$61.75. How many of each type of coin does he have?

Let x = number of quarters; y = number of dimes

$$\begin{cases} x + y = 166 \quad ① \implies x = -y + 166 \ * \\ (0.25x + 0.1y) \cdot 2.5 = 61.75 \end{cases}$$

substitute * into ②: $0.25(-y+166) + 0.1y = \frac{61.75}{2.5}$

$$-0.25y + 41.5 + 0.1y = 24.7$$
$$-0.15y = -16.8$$
$$y = 112$$

$$x = -112 + 166 = 54$$

There are 54 quarters and 112 dimes.

Ex 3. At Taco town, a taco contains 1 oz of meat and 2 oz of cheese, while a burrito contains 2 oz of meat and 3 oz of cheese. In 1 hour, the cook used 181 oz of meat and 300 oz of cheese making tacos and burritos. How many of each were made?

Let x = # of tacos; y = # of burritos:

$$\begin{cases} 1 \cdot x + 2 \cdot y = 181 \quad ① \implies x = 181 - 2y \ * \\ 2 \cdot x + 3 \cdot y = 300 \quad ② \end{cases}$$

$$2(181 - 2y) + 3y = 300 \implies 362 - y = 300 \implies y = 62$$

$$x = 181 - 2 \cdot 62 = 181 - 124 = 57$$

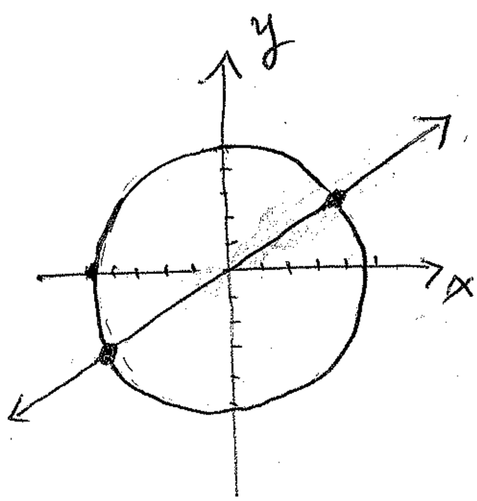
The cook made 57 tacos & 62 burritos.

8.3. Nonlinear Systems of Equations

- A nonlinear equation is an equation whose graph is not a straight line. For example, $y=3x^2$, $y=\sqrt{x}$, $y=\log x$. are nonlinear equations.
- If a system consists of at least one nonlinear equation, it's called a nonlinear system of equations.
- The solution(s) of a nonlinear system of equations, is/are the intersection(s) on the graphs of the equations.

Ex. Solve each system of equations.

1. $\begin{cases} x^2+y^2=25 & ① \\ 3x-4y=0 & ② \end{cases}$



from ②: $y=\frac{3}{4}x \quad \star$

Substitute $\star$ into ①: $x^2+\left(\frac{3}{4}x\right)^2=25$

$$\frac{25x^2}{16}=25$$

$$x^2=16$$

$$x=\pm 4$$

Use $x=\pm4$ into $\star$: $y=\frac{3}{4}\cdot 4$ or $y=\frac{3}{4}\cdot(-4)$

$y=3$ or $y=-3$

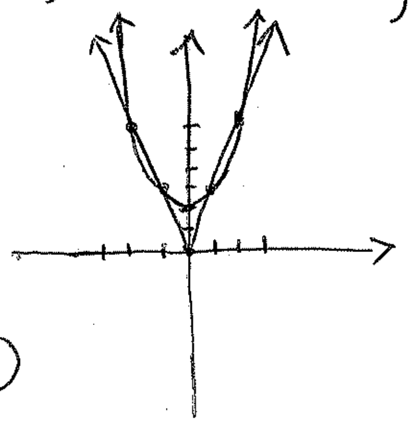
Solutions: $\{(4,3),(-4,-3)\}$

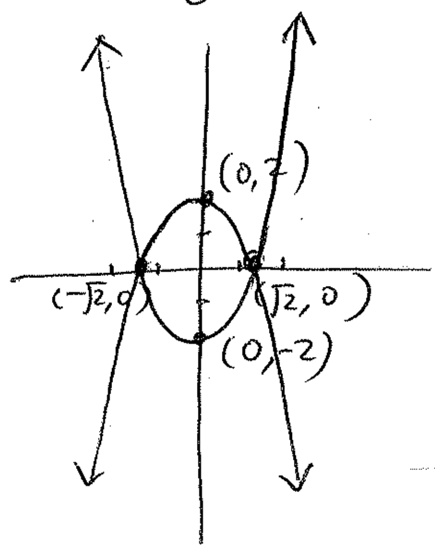
ex: (omit)
$$\begin{cases} y = 2 - x^2 & (1) \\ y = x^2 - 2 & (2) \end{cases}$$

substitute (1) into (2):
$$2 - x^2 = x^2 - 2$$
$$4 = 2x^2$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

Use $x = \pm\sqrt{2}$ in (1): $y = 2 - (\sqrt{2})^2 = 0$, or $y = 2 - (-\sqrt{2})^2 = 2 - 2 = 0$

Solutions: $\{(\sqrt{2}, 0), (-\sqrt{2}, 0)\}$.





ex 2:
$$\begin{cases} y = |3x| & (1) \\ y = x^2 + 2. & (2) \end{cases}$$

Substitute (1) into (2):

$|3x| = x^2 + 2 \Longrightarrow$

$3x = x^2 + 2$ or $3x = -(x^2 + 2)$

$x^2 - 3x + 2 = 0$ $\quad$ $-x^2 - 3x - 2 = 0$

$(x-1)(x-2) = 0$ $\quad$ $x^2 + 3x + 2 = 0$

$x = 1$, or $x = 2$ $\quad$ $(x+1)(x+2) = 0$

$x = -1$ or $x = -2$

Use $x = \pm 1$, $x = \pm 2$ in (2):

$y = 1^2 + 2 = 3$; $y = (-1)^2 + 2 = 3$; $y = (2)^2 + 2 = 6$; $y = (-2)^2 + 2 = 6$

Solution: $\{(1, 3); (-1, 3); (2, 6); (-2, 6)\}$

Ex 3: $\begin{cases} y = \sqrt{x} & (1) \\ y = 2x & (2) \end{cases}$

Substitute (1) into (2) : $\sqrt{x} = 2x$

$x = 4x^2$

$4x^2 - x = 0$

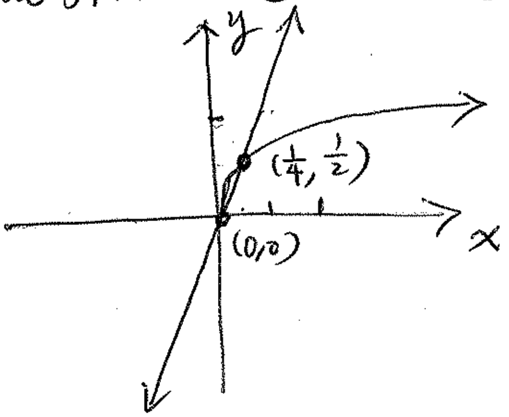
$x(4x-1) = 0$

$x = 0, \quad x = \frac{1}{4}$

use $x=0$, $y = 2 \cdot 0 = 0$

use $x = \frac{1}{4}$, $y = 2 \cdot \frac{1}{4} = \frac{1}{2}$

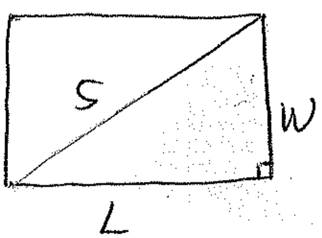
Solution: $\{(0,0), (\frac{1}{4}, \frac{1}{2})\}$.



Ex 4: A rectangular GPS monitor has a viewing area of 12 in^2, and a diagonal measure 5 in.

a) find the dimension of the monitor

b) find the Perimeter of the monitor.



$\begin{cases} L^2 + W^2 = 5^2 \\ L \cdot W = 12 \end{cases} \Rightarrow L = \frac{12}{W}$

$\left(\frac{12}{W}\right)^2 + W^2 = 25$

$\frac{144}{W^2} + W^2 = 25$

$W^4 - 25W^2 + 144 = 0$, Let $x = W^2$

$x^2 - 25x + 144 = 0$

$(x-9)(x-16) = 0$

$x = 9 \quad x = 16 \Longrightarrow W = \pm 3, \quad W = \pm 4$

if $W = 3$, $L = 4$; if $W = 4$, $L = 3$

Dimension: $\boxed{3\text{ in} \times 4\text{ in}}$ Perimeter: $2(3+4) = \boxed{14\text{ in}}$