Section 2.1. Functions

- A function is a rule that relates two sets of quantities, inputs and outputs, such that the same input cannot correspond to different outputs.
- The domain of a function is the set of all possible inputs.
- The range of a function is the set of all possible outputs.

Note, A function can appear in the form of ordered pairs, tables/lists, graphs and equations; but not all ordered pairs, tables/lists, graphs and equations are functions.

Ex.1. Determine if the following ordered pairs are functions. For functions, state the domain and range.

a) $\{(1,3),(2,3),(4,9)\}$ b) $\{(9,-3),(9,3),(4,2),(0,0)\}$

Ex.2. Determine if the following tables are functions.

a)

x	y
$1-5$	9.40
$5-10$	8.75

b)

x	y
-1	1
1	1
-5	1
5	1

When determine if a graph is a graph of a function, use the vertical line test:

If a vertical line crosses a graph more than once, then the graph is *NOT* the graph of a function.

Give some examples of graphs and have students to determine if these graphs are graphs of functions or not.

- Functions can be written as equations, such as $y = x^2 - 9x - 12$. We often replace y with a function notation, $f(x)$.
- Ex. Find the domain, and write it in interval notation for each of the following functions.

  a)$f(x) = \frac{1}{x-3}$

  b)$g(x) = \sqrt{3-x} + 5$

  c)$f(x) = -4x - 6$

Evaluate Functions

Ex. A function f is given by $f(x) = 2x^2 - x + 3$. Find each of the following.

a) $f(0)$

b) $f(-7)$

c) $f(5a)$

d) $f(x+h)$

Difference Quotient

The expression

$$\frac{f(x+h)-f(x)}{h}$$

is called the difference quotient.

Ex. Find the simplify the difference quotient for the following functions.

- a) $f(x) = -x^2 + x - 2$
- b) $k(x) = \frac{2}{x-1}$
- c) $g(x) = 2\sqrt{x+2}$ (rational the numerator in your answer)

Section 2.2 Graphs of Relations and Functions

Informal definitions of increasing, decreasing and constant functions.

- On a giving interval, if a function rises from left to right, the function is an increasing function.
- On a giving interval, if a function drops from left to right, the function is an decreasing function.
- On a giving interval, if a function stays the same value from left to right, the function is a constant function.

Note: intervals of increase, decrease and constant are open intervals.

To graph a function, we can generate a list of points, plot the points, and connect a curve through them.

Ex. Graph the following functions, find the domain and range, interval of increase, decrease and constant.

1) Constant function: $f(x) = 1$

2) Identify function: $f(x) = x$

3) Quadratic function:$f(x) = x^2$

4) Cubic function: $f(x) = x^3$

5) Absolute-Value function: $f(x) = |x|$

6) Square-Root function: $f(x) = \sqrt{x}$

7) Cube-Root function: $f(x) = \sqrt[3]{x}$

8) Semi-circle: $f(x) = \sqrt{1 - x^2}$

See page 222-223 of your textbook for a similar summary of the above basic functions and their graphs.

piecewise functions

A piecewise function uses different output formulas for different parts of the domain.

Ex. Graph each function and state the domain, range, intervals of increase, decrease and constant.

1). $$f(x) = \begin{cases} 4, & \text{for } x \le 0 \\ 4 - x, & \text{for } 0 < x < 2 \\ 2x - 6, & \text{for } x > 2 \end{cases}$$

2). $$f(x) = \begin{cases} x + 5, & \text{for } x \le -3 \\ \sqrt{9 - x^2}, & \text{for } -3 < x < 3 \\ -x + 5, & \text{for } x \ge 3 \end{cases}$$

Section 2.4 Operations with Functions

Definition

The four operations of functions:

For two functions, f and g,

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(f \cdot g)(x) = f(x) \cdot g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, for $g(x) \neq 0$.

Ex.1. Let $f(x) = 3\sqrt{x} - 2$ and $g(x) = x^2 + 5$. Find the simplify the following.

a) $(f + g)(4)$ b) $(g - f)(x)$ c) $(f \cdot g)(0)$ d) $\left(\frac{f}{g}\right)(9)$

Ex.2. Let $f(x) = \sqrt{x}$, $g(x) = 3x + 1$, and $h(x) = \frac{1}{x-4}$. Find each function and state its domain.

a) $f + g$ b) $\frac{g}{f}$ c) $h \cdot f$ d) $g - h$

Composition of Functions

Definition

If f and g are two functions, the composition of f and g, written as $f \circ g$, is defined by the equation

$$(f \circ g)(x) = f(g(x)),$$

provided that $g(x)$ is in the domain of f;

The composition of g and f, written as $g \circ f$, is defined by the equation

$$(g \circ f)(x) = g(f(x)),$$

provided that $f(x)$ is in the domain of g.

Ex.1. Let $f(x) = \sqrt{x}$, $g(x) = 2x - 1$ and $h(x) = x^2$. Find the value of each expression.

a)$(f \circ g)(5)$ b) $(g \circ f)(5)$

c) $(h \circ g \circ f)(9)$ d) $(g \circ g)(-2)$

Ex.2. Given the above function, f, g and h, find and state the domain of the following functions.

a) $f \circ g$ b) $g \circ f$ c) $h \circ g$ d) $h \circ g \circ f$

Write a function as composition

Ex.1. Let $f(x) = \sqrt{x}$, $g(x) = x - 3$, and $h(x) = 2x$. Write each given function below as a composition of appropriate functions chosen from f, g and h.

a) $F(x) = \sqrt{x-3}$ b) $G(x) = x - 6$ c) $H(x) = 2\sqrt{x} - 3$

Ex.2. If w equal to the sum of x and 16, z is the square root of w, and y is z divided by 8, then write y as a function of x.

Section 2.5: Inverse Functions

What is an inverse relation?

- When we interchange the inputs and outputs of a function, we get an inverse relation.
- the graphs of a function and its inverse are reflections of each other respect to the line

$$y = x.$$

- Note: The inverse of a function is not necessarily a function!

One-to-One Function

- Definition

  A function is one-to-one (1-1) if each output only corresponds to one unique input.

  - The inverse of a 1-1 function is also a function.

How to determine if a function is 1-1?

- the Horizontal-line Test.

  If a horizontal line can intersect the graph of a function more than once, then the function is NOT $1-1$.

How to determine if two functions are inverse of each other?

- The function f and g are inverses of each other if and only if
  1. $g(f(x)) = x$ for every x in the domain of f; and
  2. $f(g(x)) = x$ for every x in the domain of g.
  - Ex.1. Using composition, determine whether $f(x) = x^3 - 1$ and $g(x) = \sqrt[3]{x+1}$ are inverse functions.
  - Ex.2. How about $f(x) = 2x - 1$ and $g(x) = \frac{x-1}{2}$?

Find the Inverse of a $1-1$ Function $f(x)$

- 1. Interchange x and y in the given function.
- 2. Solve for y.
- 3. Replace y with $f^{-1}(x)$.

Note: if f is an one-to-one function, the domain of f is the range of f^{-1} and the range of f is the domain of f^{-1}.

Ex. For each of the 1-1 function below, find the formula for its inverse function.

a) $h(x) = 2x - 3$

b) $g(x) = \frac{2x+1}{x-3}$

c) $k(x) = \sqrt{x-9} + 5$