11.1 Sequences and Arithmetic Sequences

A sequence is a collection of things (usually numbers) that are ordered.

- A sequence goes on forever is called an infinite sequence. Otherwise, it's called a finite sequence.
- For example: $\{1, 3, 5, 7, 9, \cdots\}$ is an infinite sequence. $\{1, 3, 5, 7, 9, 11\}$ is a finite sequence.
- Each entry in a sequence is called a term. The nth term is called the general term, and n must be positive.

Ex.1. Consider the sequence whose nth term is given as $a_n = \frac{(-1)^{n-1}2^n}{n}$, and find the first 4 terms.

Ex.2. For each of the following sequences, predict the general term.

i) $1, \sqrt{3}, \sqrt{5}, \sqrt{7}, 3, \dots$ ii) $3, -9, 27, -81, \dots$ iii) $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10} \dots$

- An arithmetic sequence is a sequence such that there exists a common difference, d, between any two consecutive terms.
- The common difference: $d = a_n - a_{n-1}$.

Ex.1. Determine if each of the following is an arithmetic sequence. If so, identify the first term, a_1, and the common difference, d.

a) $34, 27, 20, 13, 6, -1, -8...$

b) $2, 4, 8, 16, ...$

c) $2, \frac{5}{2}, 3, \frac{7}{2}, 4, \frac{9}{2}...$

- The general term of an arithmetic sequence is given by

$$a_n = a_1 + (n-1)d,$$

for any integer $n \geq 1$.

- Ex.1. For the arithmetic sequence $4, 9, 14, 19, \ldots$. Write a formula for the nth term, then find the 100th term.
- Ex.2. For the arithmetic sequence $4, 7, 10, 13, \ldots$, which term is 271? That is, find n if $a_n = 271$.
- Ex.3. Find a_1 when $d = 4$ and $a_8 = 33$.
- Ex.4. Find the common difference of an arithmetic sequence whose first term is 5 and the 11st term is -10.

11.3 Geometric Sequences and Series

- A geometric sequence is a sequence such that there exists a common ratio, r, between any two consecutive terms.
- For example, $\{1, 2, 4, 8, 16, 32, ...\}$ and $\{a, ar, ar^2, ar^3, ar^4, ...\}$, are geometric sequences.
- The general term of a geometric sequence is given by

$$a_n = ar^{n-1}, \qquad \text{where } r \neq 1 \text{ and } r \neq 0$$

- The common Ratio can be found: $r = \frac{a_n}{a_{n-1}}$

Ex.1. Identify the common ratio.

a) $5, \frac{5a}{2}, \frac{5a^2}{4}, \frac{5a^3}{8}, ...$ b) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, ...$

- Ex.1. Find the general term and then the 9th term of the geometric sequence $\frac{1}{64}, \frac{1}{-32}, \frac{1}{16}, \frac{1}{-8}, \ldots$
- Ex.2. Find the general term and then 502th term of the geometric sequence $-1, 1, -1, 1, \ldots$.
- Ex.3. Write a formula for the nth term of each geometric sequence.

  i) $\frac{1}{6}, 0.5, 1.5, 4.5, \ldots$ ii) $2, 0.2, 0.02, 0.002, \ldots$
- Ex.4. Identify each sequence as arithmetic, geometric or neither.

  1) $0, 2, 4, 6, 8, \ldots$ 2) $\frac{1}{6}, \frac{1}{3}, 1, 4, \ldots$ 3) $5, 1, \frac{1}{5}, \frac{1}{25} \ldots$

Sum of an Infinite Geometric Series

- If $a + ar + ar^2 + ar^3 + ...$ is an infinite geometric series with $|r| < 1$, then the sum S of all the terms is given by

$$S = \frac{a}{1 - r}$$

Ex. Find the sum of the following geometric series, if possible.

i) $1 + \frac{1}{3} + \frac{1}{9} + ...$

ii) $3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - ...$

iii) $-1 + 3 - 9 + 27 - ...$