3. Games of Strategy

3.1. Two Player Zero-Sum Games

In the Chapter, we will always assume that there are only two players, and that the set of possible outcomes for each player is the set of all real numbers.

3.1.1. Definition.

A two-player game such that the set of each possible outcomes for each player is the set of all real numbers is called a zero-sum games, if for each leaf L of the game tree we have

$$\omega(L) = (r, -r)$$

If we just care whether a player wins or looses, then the possible outcomes are either 1 (for winning), 0 (for a draw) and -1 (for loosing). In other cases, we might count the number of points a player gains. If total number of points that is available for both players together is a fixed number c, then for each leaf we have

$$\omega(L) = (r, s)$$

with $r + s = c$. Using the definition, such a game would be a zero-sum game only if $c = 0$. If c is not equal to zero, then we have to make a slight modification to turn this game into a zero-sum game. If $\omega(L) = (r, s)$ with $r + s = c$, we define

$$\omega_c(r, s) = \left(r - \frac{c}{2}, s - \frac{c}{2}\right)$$

and the game with the new outcome function ω_c will be a zero-sum game.

Other important features of the games we are discussing at the point include the following properties:

1. There are no chance moves.
2. Every player has complete information at each point of the game.
3. The games are strictly competitive. The gain of one player equals the loss of the other player.

Many games cannot end in a draw, i.e. 0 will not be a possible outcome for the game. For other games, like chess or checkers, this is not the case.