Mathematics 121 – Game Theory

Homework Assignment No. 6

1) A randomized bimatrix game (A, B) is given by the following matrices:

$$A = \begin{pmatrix} 2 & 3 \\ 5 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix}$$

Find a point of equilibrium.

2) Repeat the previous problem if

$$A = \begin{pmatrix} 2 & 3 \\ 5 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 3 & 2 \\ 6 & 3 \end{pmatrix}$$

3) Use the idea of row and column domination to find the equilibrium of the randomized bimatrix game given by

$$A = \begin{pmatrix} 4 & 4 & 6 & 1 & 4 & 1 \\ 3 & 1 & 2 & 0 & -1 & 3 \\ 1 & -1 & -1 & 1 & 4 & 5 \\ -1 & 2 & 2 & 1 & 1 & 7 \end{pmatrix}$$

$$B = \begin{pmatrix} -2 & 3 & 5 & -1 & 2 & -1 \\ 1 & 2 & 5 & 3 & 3 & 2 \\ 2 & 3 & 5 & 2 & 3 & 2 \\ 3 & 5 & 5 & 1 & 2 & 0 \end{pmatrix}$$

4) It's raining again, and Abigail and Horace are inventing another gain. This time it's a version of Battleship. They are playing this game on a 4×1 board. Horace occupies to adjacent squares with his ship, and Abigail drops two bombs. Nothing serious, just peas. She can select two of the four fields on which she would like to drop the peas. But both have different goals. Horace, knowing that he probably cannot avoid being hit, is happy if his ship is not sunk (i.e. if Abigail does not score two hits). He gets 1 point if his ship survives and 0 points if his ship is sunk Abigail, on the other hand, just wants to maximize the number of hits. She gets one point for each hit.

   a. Represent this game as a bimatrix game. Abigail's pure strategies are the 6 ways she can select two of the four fields to drop the peas. Horace's strategies are the three different ways to occupy to adjacent fields.

b. Show that the resulting bimatrix game has no equilibrium consisting of pure strategies.
c. Use row domination to reduce this bimatrix game to a smaller game (with three rows and three columns)
d. Randomize the reduced game and find at least one point of equilibrium.
e. What are the expected outcomes for the point of equilibrium found in (d)?

10% extra credit will be given to all students with the highest expected values, either for Abigail or Horace. These 10% will be added to the total score of the course. You have to submit a written solution of the problem in order to be eligible for the extra credit.