2.2. Strategies

In order to describe strategies, we need some additional notations

2.2.1. Definition

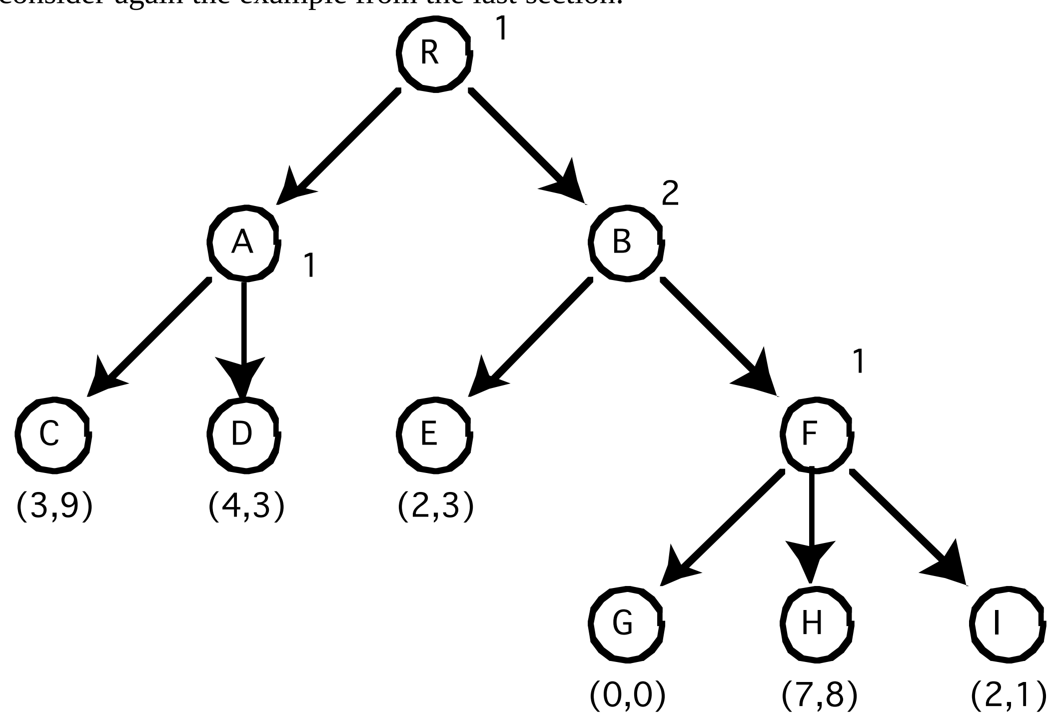
Let t be a tree and let n,m be two nodes of t. If there is an edge from n to m, then n is called the parent of m and m is called a child of n.

In particular, the root has no parent and a leaf has no children.

2.2.2. Definition

Let G be an n - player game.. A strategy σ^i picks *for player i picks for each node where it is player's i turn to move a child of that node.*

Formally, a strategy for player i is a function σ^i that picks for each node n labeled with i a child m of that node n. These functions can be given as a set of ordered pairs of node. If we consider again the example from the last section:



then strategies for players 1 and 2 are given by the following two lists:

$$\sigma^1 = \{(R,A),(A,D),(F,H)\}$$

$$\sigma^2 = \{(B,E)\}$$

Those are not the only possible strategies. Also, we do not require that strategies lead to optimal results. In this case, if both players follow their strategy, the game would end at node D.

We now have to formalize what the outcome of a game is, once the strategies are fixed. In order to do this, we have to assume that the game tree is finite. Hence this defintion will not work for Parcheesi or other infinite game tree.

We start with the root R. If it is player *i*th turn to move, then this player will move to the node $N_1 = \sigma^i(R)$. We now continue recursively: If the node N_k is already defined, and if that node is not a leaf, then a certain player with index j_k gets to play at this node, and we define

a. $$N_{k+1} = \sigma_j(N_k)$$

Since the game tree is assumed to be finite, this procedure will end at a leaf. This leaf is called the outcome of the game under

2.2.3. Definition.

The leaf from the previous construction is called the outcome of the game under the strategies $\sigma_1, \cdots \sigma_n$ *and denoted by*

b. $$\omega(\sigma_1, \cdots \sigma_n)$$

If we are dealing with a two-player game, then all possible strategies of the game together with all possible outcomes can be listed in a table (matrix). This matrix is frequently also called the **strategic form of the game.** We illustrate this with with the game tree form the previous example.

We use the following table to list all strategies and all outcomes:

			B	E	F
R	A	F			
A	C	G		(3,9)	(3,9)
A	C	H		(3,9)	(3,9)
A	C	I		(3,9)	(3,9)
A	D	G		(4,3)	(4,3)
A	D	H		(4,3)	(4,3)
A	D	I		(4,3)	(4,3)
B	C	G		(2,3)	(0,0)
B	C	H		(2,3)	(7,8)
B	C	I		(2,3)	(2,1)
B	D	G		(2,3)	(0,0)
B	D	H		(2,3)	(7,8)
B	D	I		(2,3)	(2,1)

For instance the very first column of the table shows the action of the first player: He moves from R to A (first 6 rows) or to B (last 6 rows). In the second column we find the moves of the first player if node A is reached.

The very first row shows that the second player can move from B either to E or to F.

The ordered pairs show the outcomes of the game if the corresponding strategies are uses.