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Mathematics 121 – Game Theory

Homework Assignment No. 4

1. Here is a game that you can play with your elderly relations. It's called "Frogs and Toads", and is being played on a board that has one column of 19 fields. The players control one token each (a frog or a toad), and put their token at opposite ends of the board. Frogs and Toads can move only forward, 1 or 2 squares at a time. They cannot hop over each other, they cannot change direction, and they cannot hop on a field that is already occupied by a different token. The last player who was able to move wins the game. The elderly relation plays as "Toads" and starts the game. You play as "Frogs".
    a. Who wins the game, the old toads or the young hoppers?
    b. Who wins the game, if both frogs and toads can hop as many squares forward as they chose?
    c. Who wins the game, if the amphibians can move either 1 or 2 fields forward or 1 or 2 fields backwards?
    d. Who win the game, if they can move forward and backwards as many squares as the chose?
    e. How do the answers change, if the game is played on a board with 3 columns, where the first column has 19 fields and each of the two other columns have 17 fields? In this case, each player controls 3 amphibians,

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and neither frogs nor toads change the column they are in.

2. We consider the version of Nim, where each player can remove either 1 or 2 matches from a pile of n matches. Analyze the game when the players do not alternate in moving but always toss a fair coin to decide who moves next.
   a. If $n = 3$.
   b. If $n = 5$.
3. It's raining. Abigail and Horace decide to turn off the TV and play a game. They get three boxes and place them on the table. Abigail hides a coin in one of the boxes. Horace then has to guess where she hid the treasure. He points at one of the boxes. Abigail – who knows which box contains the coin – opens one of the remaining two boxes. Horace now has the opportunity to change his mind. Horace wins if he finds the treasure. In the beginning, Horace tries to read Abigail's facial expression before he makes a decision. Of course, she fouls him completely. So Horace decides to make his randomize his moves completely.
   a. Draw the game tree.
   b. What is Abigail's optimal strategy?
   c. What is the probability for Horace to win the game?
   For parts (d) and (e), Abigail will stick to her optimal strategy. After all, she does not want to make it easy for Horace.
   d. If Horace never changes his mind, explain why his chance of winning the game is now 1/3.
   e. If Horace always changes his mind, explain why his chance of winning the game is now 2/3.
   f. Find a partner to play this game. Are your predictions really correct? Don't wait for the rain, we are in Southern California!
4. For the following compound lottery, find the probability distribution at the leaves. If a random variable takes the values indicated at the leaves, what is the expected

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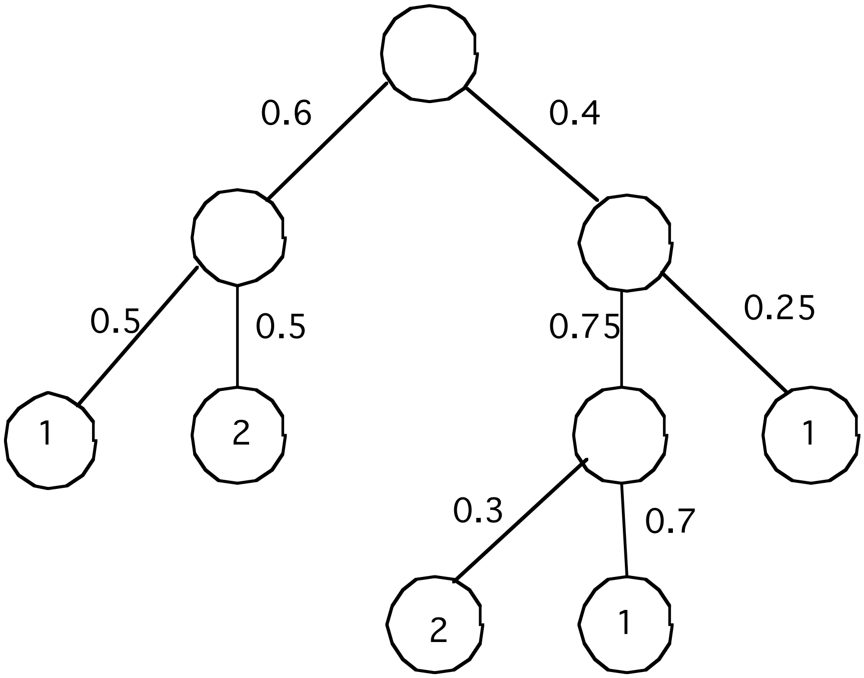
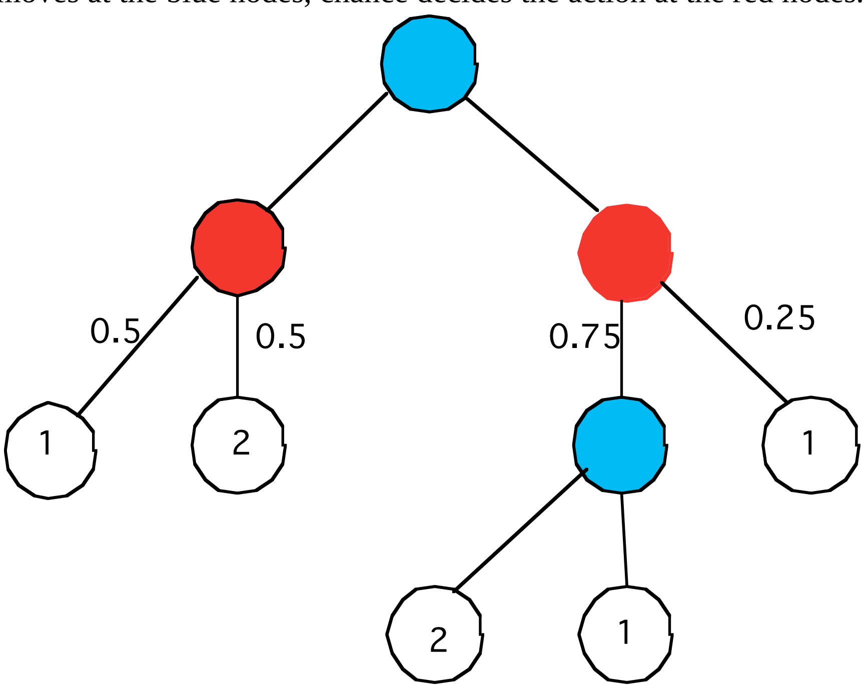
value of the lottery?



5. For the following one-player game tree with chance moves, find the optimal strategy for the player. What is the expected value of the game? (The player moves at the blue nodes, chance decides the action at the red nodes.)



6. For the following zero-sum two-player game tree with chance moves, find the best strategy for both players. What is the expected value of the game? (Player 1

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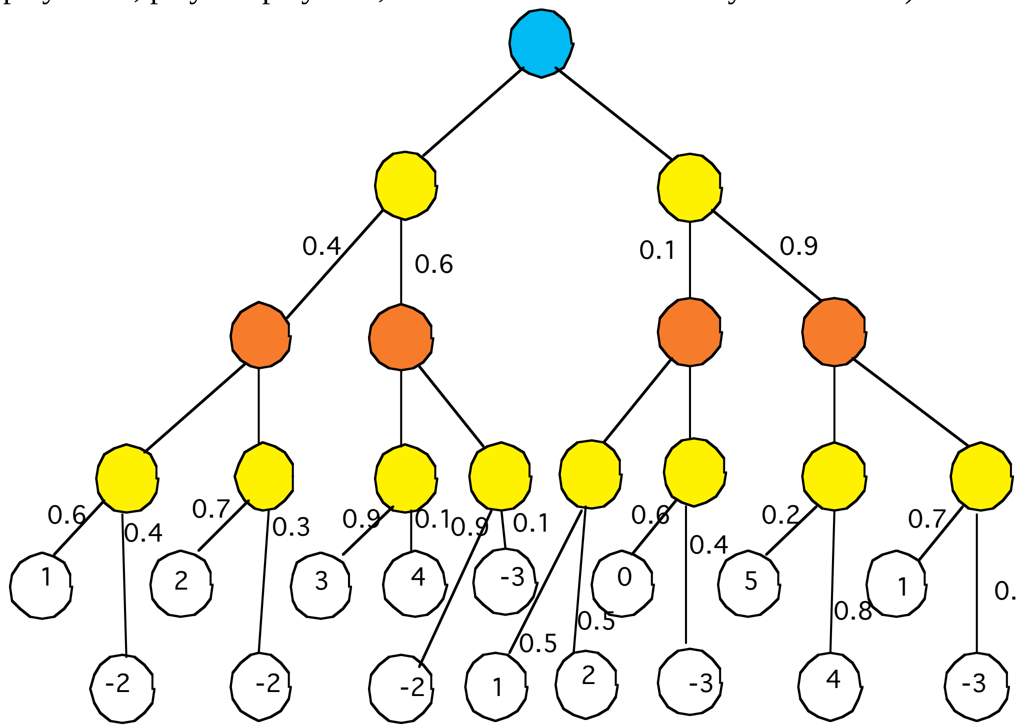
plays blue, player 2 plays red, and chance decides at the yellow nodes.)



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