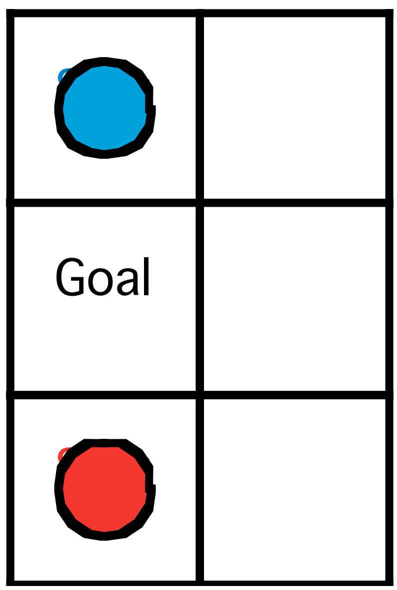
1.2. Miniature Parcheesi

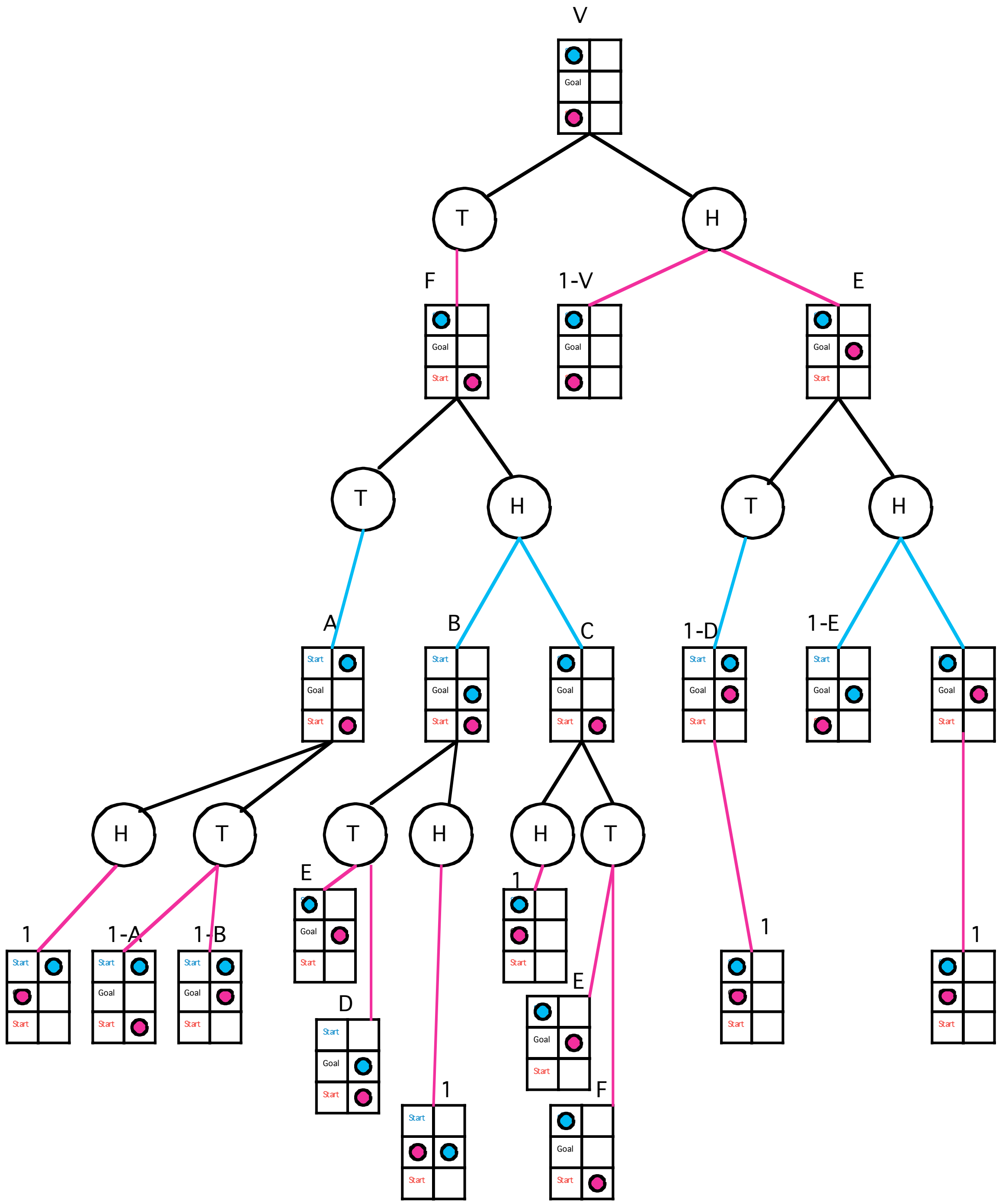
As a second example, we would like to analyze the game of Parcheesi. Of course, this game would be much too big. Following the ideas of Ken Binmore's book "Fun and Games", we will use the following tiny board and play by flipping a coin instead of rolling a dye.

Here are the rules of our minitare game:

After each flip of the coin, we have two option: Either move or pass. If a head was flipped, we move two squares, and if a tail was the outcome of the coin flip, we move one square.
The first player to move the goal wins. If are directly in front of the goal and a head was flipped, we still can move into the goal, even though we have to move two squares.



We draw again the game tree. This time, luck plays a role, so there are extra lines representing the coin flips:



The first thing to observe that this is really an infinite tree. For instance, the letter F occurs in two different spots, where we have reached the same game situation. We need both node, because luck might determine a different path the next time around. So

We are inserting a tree into itself!

Later, this will result into a recursive definition a trees that do the same thing mathematically.
Also, if the pink player chooses not to move in the beginning, then the bleu player becomes the first player in the game situation reverses. This is reflected by the fact that the root has the latter V attached, while the node immediately following the root representing the same game situation, except that now bleu starts out, has the letter 1-V attached. This also happens in various other places.

The letters now represent the probability that the pink player wins. This also gives meaning to the labeling with 1-V etc. If pink starts and has probability V to win, then – if bleu starts –she has probability 1-V to win (since now blue wins with probability V).
Again, this argument works for various other nodes, too.
In certain case, it is clear that pink will win, no matter what the outcome of the coin toss is. Those nodes are labeled with 1.

We now use Zermelo's algorithm again. When it is pink's turn to move, she will always try to move in the direction where the probability to win is high. Bleu will do the opposite – he will move in the direction where her probability to win is small. This explains the formation of maxima and minima in the following formulas.
We now find the following equations:

$$V = \frac{1}{2}F + \max\{1-V, E\}$$

$$A = \frac{1}{2} + \frac{1}{2}\max\{1-A, 1-B\}$$

$$B = \frac{1}{2} + \frac{1}{2}\max\{E, D\}$$

a. $$C = \frac{1}{2} + \frac{1}{2}\max\{E, F\}$$

$$D = 0$$

$$E = \frac{1}{2}(1-D) + \frac{1}{2}\min\{1-E, 1\}$$

$$F = \frac{1}{2}A + \frac{1}{2}\min\{B, C\}$$

Solving those equation leads to

$$A = \frac{2}{3}$$

$$B = \frac{5}{6}$$

$$C = \frac{7}{8}$$

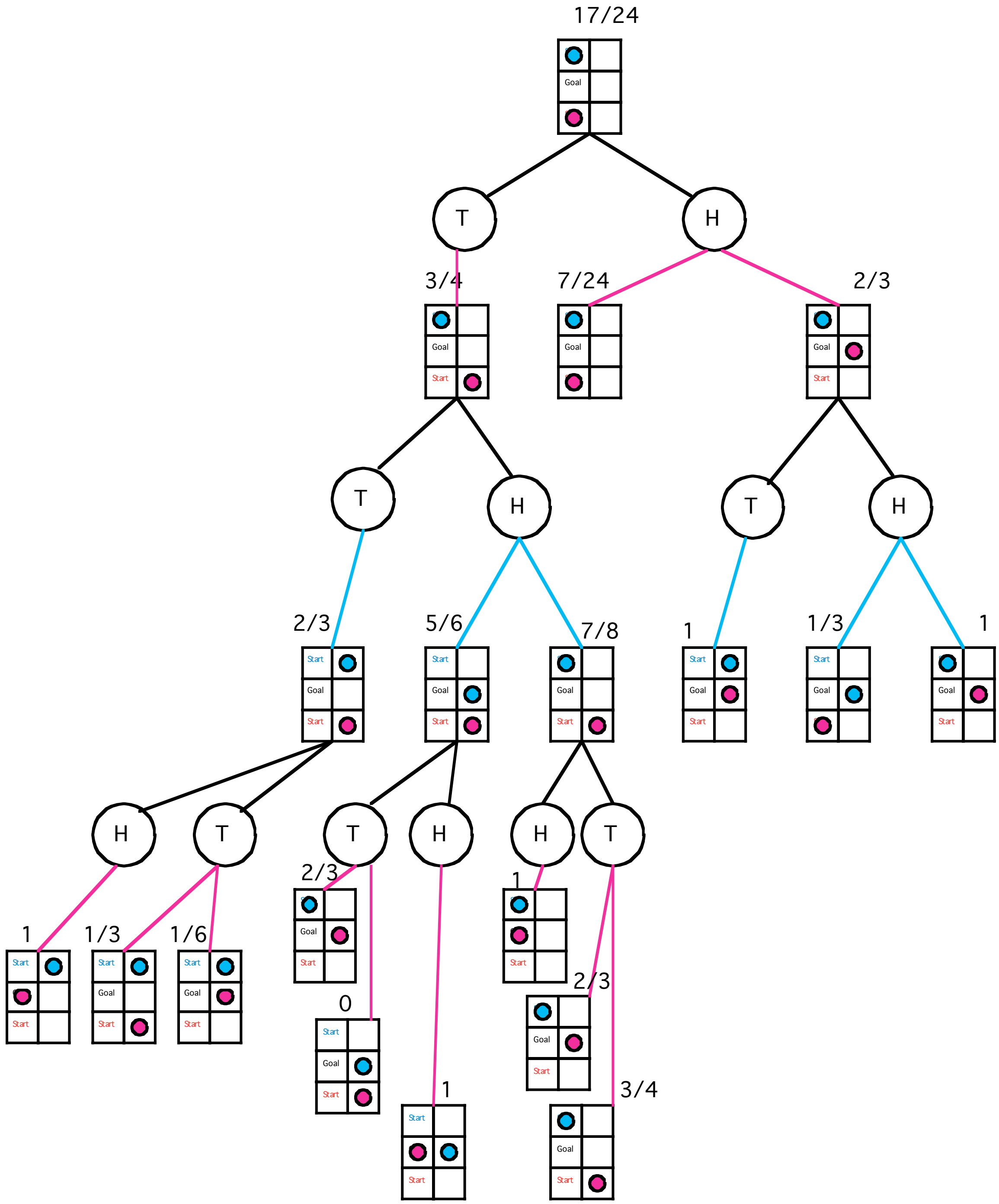
b. $$D = 0$$

$$E = \frac{2}{3}$$

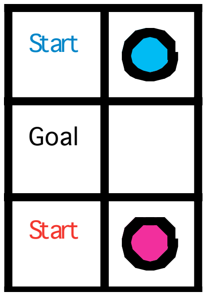
$$F = \frac{3}{4}$$

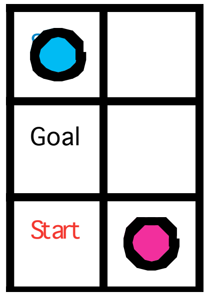
$$V = \frac{17}{24}$$

If we look at the diagram again with those numbers, we find



Studying the tree, we find that the pink player should always move, unless tail is shown in one of the following two positions:





c. The pink player will win with probability 17/24