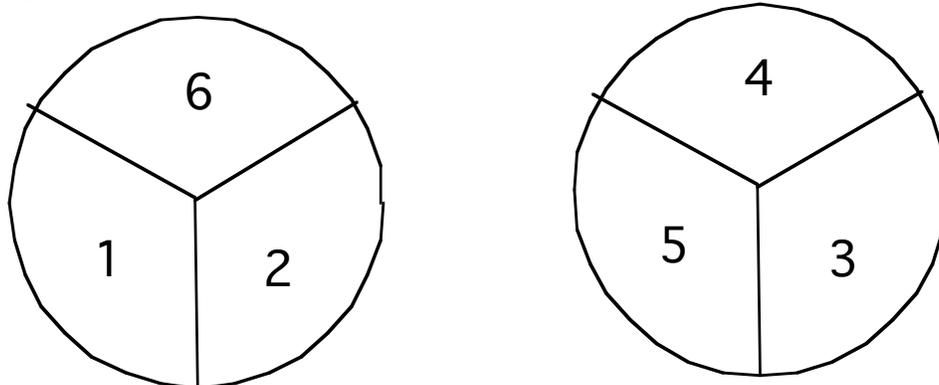


## Mathematics 121 – Game Theory

### Homework Assignment No. 3

The game of Nim is played as follows:  $n$  piles of matches are placed on a table. Two players take turns to remove the matches. Each player chooses one of the piles and takes as many matches from this pile as he or she likes. The player who removes the last match from the table wins the game.

1. Describe a play of the game of Nim with 3 piles of matches, in which the first pile contains 1 match, the second pile contains 2 matches and the 3 pile contains 4 matches.
2. Who would win the game of Nim with  $n > 1$  piles of matches of which the  $k$ th pile contains  $2^{k-1}$  matches.
3. Suppose that you are dealt the A, K, Q and 10 of hearts and the 2 of clubs. In Draw Poker, you get to change some of your cards after the first round of betting. If you discard the 2 of clubs, hoping to draw the J of hearts, what is the probability that you will be successful?  
What is the probability of drawing a straight? Any J will suffice for this purpose.
4. A box contains one gold and two silver coins. Two coins are drawn at random from the box. A man looks at the coins that have been drawn without your being able to see. He then selects one of the coins and shows it to you. It is silver. What is the probability that the other is gold? What is the probability that the other coin is gold, if the coin that you are shown is selected at random from the drawn pair? (Hint: The answers are  $2/3$  and  $1/2$ )
5. When a “roulette wheel” from the following figure is spun, each number is equally likely to result.



Two players compete. Player 1 chooses one of the wheels, and the second player gets the remaining wheel. Then both players spin their wheels simultaneously.

- a. The player whose wheel stops on the higher number wins. If you are player 1, which wheel would you choose? What would be your probability of winning the contest?
- b. Now the player whose wheel stops on the higher numbers wins the total number of points showing on both wheels, and the second player receives 0 points. (For example, if the first wheel stops at 6 and the second wheel stops at 4, the player who operates the first wheel wins 10 points, and the other player wins nothing.) If you are operating the first wheel, how many points would you expect to win? How many points would opponent expect to win?