Mathematics 121 – Game Theory

Homework Assignment No. 2

1. Everybody knows the game of tic-tac-toe on a three-by-three board.
    a. Does the first player win or loose? Or are there optimal strategies so that the game ends at least in a draw?
    b. Find an optimal strategy for the first player.
    c. Invent rules for tic-tac-toe, played on a four-by-four board. Find a partner to play this game a couple of times and improve the rules until the games become more interesting.
    d. Answer question (a) for your rules on a four-by-four board.
2. A simplified version of the game of Nim has the following rules. A number of n matches are placed upon a table. Players alternatively take either *1* or *2* matches from the pile. Whoever takes the last match wins the game.
    a. Let t_n be the game tree for a game starting with n matches, not including the labeling. If $n > 2$, show that $t_n = [t_{n-1} \oplus t_{n-2}]$
    b. Find the game trees for $n = 1$ and $n = 2$.
    c. Use the recursion to find the game trees t_n for $n = 1,2,3,4$ and *5*, including the labeling by the players. Also, indicate at the leaves who is winning or loosing the game.
    d. Carry out Zermelo's algorithm and list all winning strategies for $n = 5$. Is there more than one winning strategy?
    e. Use the recursion to find an explicit formula for the number of nodes (including the leaves) for the game tree t_n
3. Using the theorem about maximinis and mimimaxes, which of the following matrices could occur as the table in the strategic form of a two-player zero-sum game?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 4 & 2 \\ 7 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

4. Is there a two-player game so that each player has 2 strategies and so that the game tree has exactly 9 leaves? Can the matrix B from the previous problem indeed be the matrix of a game?
5. The following tree shows a two-player zero-sum.
    a. Find all strategies for player 1 (Caribbean blue) and player 2 (pure magenta).

b. Find the strategic form of the game

c. Use the strategic form to find

   i. The value of the game.

   ii. All optimal strategies for both players.