Microeconomics

Question 1 (Microeconomics, 30 points). A ticket to a newly staged opera is on sale through sealed-bid auction. There are three bidders, Alice, Bert and Carl. Alice values the ticket at \$10, Bert at \$20, and Carl at \$30. The bidders are free to submit a bid of any positive amount.

a. (**6 points**) Show that there is no dominant strategy for any bidder if the highest bidder wins the ticket and pays his own bid.

Answer:

If the highest bidder wins the object and pays his bid, then any bid below his own valuation can be the optimal bid. For example, for Alice who has a valuation of \$10, bidding \$5 is better than \$9 if the other two bidders bid below \$5, since she wins the ticket with either bid but the lower bid means she pays less and gives her a higher payoff.

However, if the other two bidders bid between \$5 and \$9, bidding \$5 makes her to lose the ticket, while bidding \$9 makes her to win the auction and get the ticket at a price less than the valuation which yield a positive payoff.

The same logic applies to all other bids below the bidder's valuation for all three bidders. There isn't any bid which will always yield a higher payoff for the bidder than all other bids.

Points:

6 points for a well-illustrated answer, preferably with examples.

Partial credit at the grader's own judgement.

b. (**5 points**) From now on, assume this is a second-price auction, that is, the highest bidder wins the ticket and pays the second-highest bid. If everyone bids his or her own valuation, what is the payoff of each bidder?

Answer:

If everyone bids his or her own evaluation, Carl wins the object and pays \$20. Hence his payoff is $30 - 20 = \$10$. The other two bidders get 0.

Points:

2 points for Carl winning.

1 point each for the 3 bidders' payoff.

c. (**4 points**) Show that "everyone bids his or her own valuation" is a Nash Equilibrium.

Answer:

To see if this is a Nash Equilibrium, we see if any bidder wants to change their bids.

If everyone bids his or her own valuation, both Alice and Bert get a payoff of zero since they will not win the auction. If they change their bids to below \$30, they will still not win the auction; if they change their bids to higher than \$30, they will win the ticket but will have to pay at least \$30 for it, which is higher than their valuation and give them negative payoffs. Therefore both Alice and Bert have no incentive to change their bids.

Carl gets a payoff of \$10 since he values the ticket at \$30 but only has to pay \$20 for it. If he changes his bid to a value about \$20, his payoff will not change; if he changes his bid to a value below \$20, he will not win the auction and will only get a payoff of 0 which is less than what he is getting before. Since no change will give him a higher payoff, Carl has no incentive to change his bid.

Since all the bidders have no incentive to change their bids, this is a Nash Equilibrium.

Points:

2 points to show the high bidder will not deviate.

2 points to show the low bidders will not deviate.

d. (**7 points**) Now suppose that there is a large number of tickets on sale and a large number of bidders, with $\frac{1}{3}$ of them valuing the ticket at \$10, $\frac{1}{3}$ at \$20 and $\frac{1}{3}$ at \$30. Each ticket is sold at a second-price sealed-bid auction independent of other auctions, and each auction has 3 bidders attending, where each of them can be any of the three types. Knowing that each bidder will still bid his or her own valuation, what is the expected revenue from a single auction? (Note: if there is a tie, for example: $(20, 20, 10)$, the second highest bid will be 20)

Answer:

Since each of the 3 bidders can be of any type, there will be a total of $3 \times 3 \times 3 = 27$ outcomes.

To calculate the expected revenue, we need to count the number of outcomes that yields a particular value of revenue.

To get \$10, we need the second-highest bid to be \$10, hence we have the bids being: $(10, 10, 10)$ 1 case $(10, 10, 20)$ 3 cases $(10, 10, 30)$ 3 cases for a total of 7 cases.

To get \$20, we need the second-highest bid to be \$20, hence we have the bids being: $(10, 20, 30)$ 6 cases $(20, 20, 30)$ 3 cases $(20, 20, 20)$ 1 case $(20, 20, 10)$ 3 cases for a total of 13 cases.

To get \$30, we need the second-highest bid to be \$30, hence we have the bids being: $(30, 30, 30)$ 1 case $(30, 30, 20)$ 3 cases $(30, 30, 10)$ 3 cases for a total of 7 cases.

Hence the expected revenue is

$$\frac{7}{27} \times 10 + \frac{13}{27} \times 20 + \frac{7}{27} \times 30 = \$20$$

Points:

3 points for the correct method.

1 point each for each correct probability.

1 point for the correct answer.

e. (**8 points**) The auctioneer knows that if the tickets are not sold, they can be sold later for \$15 each. Therefore he sets a minimum bids of \$15 for each auction. That is, if the second-highest price is less than \$15, the highest bidder will be asked to pay \$15. Bidders may choose to bid \$0, in which case they will not win the ticket even if all the other bidders bid \$0 (the ticket will not be sold if all 3 bidders bid \$0. However, for example, if the bids are $(20, 0, 0)$, the second highest bid is 0, and the bidder who bids \$20 will win the ticket and pay \$15).

How much will each type of bidders bid in the Nash Equilibrium, and what is the expected revenue from a single auction, not counting the later sale if the ticket is not sold?

Answer:

The bidders who have a valuation of \$10 always get a payoff of 0 in (d) whether they win the auction or not. With a minimum price of \$15, they may get a negative payoff if they bid a positive value and win the auction. Hence the bidders with \$10 valuation will bid 0.

The bidders with \$20 and \$30 valuation will still bid their own valuation, by the same logic as in (c).

To calculate the expected revenue, we need to count the number of outcomes that yields a particular value of revenue. There are still a total of 27 outcomes.

The ticket is not sold if all bidders have a valuation of \$10, which is 1 case.

To get \$15, we need the second-highest bid to be 0, hence we have the bids being: $(0, 0, 20)$ 3 outcomes $(0, 0, 30)$ 3 outcomes for a total of 6 cases.

To get \$20, we need the second-highest bid to be \$20, hence we have the bids being: $(10, 20, 30)$ 6 cases $(20, 20, 30)$ 3 cases $(20, 20, 20)$ 1 case $(20, 20, 10)$ 3 cases for a total of 13 cases.

To get \$30, we need the second-highest bid to be \$30, hence we have the bids being: $(30, 30, 30)$ 1 case $(30, 30, 20)$ 3 cases $(30, 30, 10)$ 3 cases for a total of 7 cases.

Hence the expected revenue is

$$\frac{6}{27} \times 15 + \frac{13}{27} \times 20 + \frac{7}{27} \times 30 = \$\frac{560}{27}$$

Points:

4 points to explain the new bids.
2 points to note when the ticket will not be sold.
1 point for using the same probabilities as in (d).
1 point for the correct answer.

Question 2 (Microeconomics, 30 points). The demand function in a particular industry is of the form $q = \frac{100}{p^2}$, where q is the quantity demanded and p is the price. The technology in the market is such that each unit can be produced at the constant cost of \$5.

a. (**4 points**) If the industry is supplied by perfectly competitive firms, what is the equilibrium price and industry output? What is the price elasticity of demand faced by a single firm at this price if the firm increases its price?

Answer:

When the industry is perfectly competitive, we have $p = MC$. Therefore the equilibrium price is \$5, and industry output is

$$\frac{100}{25} = 4$$

For any perfectly competitive firm, if it wants to increase its price above the equilibrium price, it will lose all its customers. Therefore the price elasticity it faces is infinity.

Points:

1 point for the correct method.
1 point each for price, quantity, and elasticity.

b. (**10 points**) If the industry is supplied by a monopolist, what is the equilibrium price and industry output? What is the price elasticity of demand faced by the monopolist at this price if the firm increases its price?

Answer:

A monopolist will equate its marginal revenue to the marginal cost.
We calculate the total revenue as:

$$TR = pq = (\frac{100}{q})^{\frac{1}{2}} q = 10q^{\frac{1}{2}}$$

And marginal revenue is:

$$MR = \frac{dTR}{dq} = 5q^{-\frac{1}{2}}$$

Equate this to $MC = 5$, we have $q = 1$, and $p = \sqrt{100} = 10$.

The price elasticity of demand is calculated as

$$\frac{dq}{dp} \times \frac{p}{q} = -200p^{-3} \times \frac{p}{100p^{-2}} = -2$$

which is constant at any position on the demand curve.

Points:
2 points for noting MR=MC.
2 points for TR.
2 points for MR.
2 points for p and q.
2 points for elasticity.

c. (**11 points**) If the industry is supplied by two identical oligopolists engaged in Cournot (simultaneous quantity setting) competition, what is the equilibrium price and industry output?

Answer:
From firm 1's point of view, the price may be expressed as:

$$p = (\frac{100}{q_1 + q_2})^{\frac{1}{2}} = 10(q_1 + q_2)^{-\frac{1}{2}}$$

Hence firm 1's total revenue is

$$TR_1 = 10q_1(q_1 + q_2)^{-\frac{1}{2}}$$

Its marginal revenue is calculated as the following, taking q_2 as given:

$$MR_1 = \frac{dTR_1}{dq_1} = 10(q_1 + q_2)^{-\frac{3}{2}}(\frac{1}{2}q_1 + q_2)$$

Since the two firms are identical, we must have $q_1 = q_2$. Substitute this into the marginal revenue expression and equate it to 5, we get

$$10(2q_1)^{-\frac{3}{2}}(\frac{3}{2}q_1) = 5$$

Solving, we get $q_1 = \frac{9}{8}$. Since $q_2 = q_1$, we must have total output being $2q_1 = \frac{9}{4}$.

And industry price is $10 \times \frac{9}{4}^{-\frac{1}{2}} = \frac{20}{3}$.

Points:
2 points for price (first equation).
2 points for TR.
2 points for MR.
1 point for noting $q_1 = q_2$.
2 points for q.
2 points for p.

d. (**5 points**) If the industry is supplied by two identical oligopolists engaged in Betrand (simultaneous price setting) competition, what is the equilibrium price and industry output? If there are n Betrand oligopolists, what is the equilibrium price and industry output?

Answer:
In Betrand competition, price will fall to the marginal cost at equilibrium. Otherwise, a firm can always cut prices to capture the entire market. Hence we will have $p = 5, q = 4$. This result is the same for any number of Betrand oligopolists.

Points:
1 point for the correct argument.
1 point each for each answer (price/quantity $\times$two firms/n firms).