Microeconomics

Instructions: Answer any three of the following four questions. Whenever possible, justify your answers. Write your answer to each question in a **separate** bluebook. Write the number of the question on the cover of the bluebook. DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam is 4 hours.

A bank has two types of borrowers: the highly productive (type h) and less productive (type l) firms. It is known that they are equally likely. The firms finance their investments entirely through a bank loan. There is only one bank the firms can borrow from. If a type h firm gets a loan of size k, it will generate a gross profit (after substracting k) of $2\sqrt{k}$ whereas a type l firm will generate a gross profit of $\sqrt{k}$. The bank has an alternative investment which would yield rk, where $r > 0$, for investment of size k. If the bank loans k and the repayment is $k+t$, the bank's 'profit' is $t - rk$. If a type h firm (resp. type l firm), borrows k and repays $k+t$, it will have net profit $2\sqrt{k} - t$ (resp. $\sqrt{k} - t$). The bank maximizes its expected profit.

1. Set up the optimal contract problem of the bank

Answer: $\max_{\{k_h,k_l,t_h,t_l\}} \frac{1}{2} * (t_h - rk_h) + \frac{1}{2} * (t_l - rk_l)$ subject to:

1. $2\sqrt{k_h} - t_h \geq 2\sqrt{k_l} - t_l$

$2\sqrt{k_l} - t_l \geq \sqrt{k_h} - t_h$

3. $2\sqrt{k_h} - t_h \geq 0$

4. $\sqrt{k_l} - t_l \geq 0$.

2. Explain why the problem you formulated in 1. is the right problem to solve.

Answer: invoke the revelation principle.

3. Show that at the optimum, the incentive constraint of the type h and the participation constraint of type l bind.

Answer: If 4 does not bind, $2\sqrt{k_h} - t_h \geq 2\sqrt{k_l} - t_l \geq \sqrt{k_l} - t_l > 0$. Then, t_h, t_l can be increased a little by the same amount, increasing the maximum. If 1. does not bind, $2\sqrt{k_h} - t_h > 2\sqrt{k_l} - t_l \geq \sqrt{k_l} - t_l \geq 0$, then t_h can be increased a little, increasing the maximum.

4. Solve the optimal contract problem and interpret it.

Answer: Since the incentive constraint of h and the participation constraint of l binds, $t_l = \sqrt{k_l}$ and $2\sqrt{k_h} - t_h = 2\sqrt{k_l} - t_l$. Then, $t_h = 2\sqrt{k_h} - 2\sqrt{k_l} + \sqrt{k_l} = 2\sqrt{k_h} - \sqrt{k_l}$., solution: $k_h = \frac{1}{r^2}, k_l = 0, t_l = 0, t_h = \frac{2}{r}$. These satisfies constraints 3, 4.

5. Now, suppose that the outcome of investment is random. A type h firm if it invests k has $\frac{1}{2}$ probability of getting a gross profit of $2\sqrt{k}$ and $\frac{1}{2}$ probability of a getting gross profit of $\sqrt{k}$. A type l firm if it invests k has $\frac{1}{4}$ probability of getting a gross profit of $2\sqrt{k}$ and $\frac{3}{4}$ probability of getting a gross profit of $\sqrt{k}$. Assume that repayment contract is not contingent on the profit outcome. Assume that the firms have unlimited liability: the firms can and must pay contracted t (as well as k) no matter the size of the profits. Set up the problem and solve it. The firms as well as the bank are risk neutral.

Answer: $\max_{\{k_h,k_l,t_h,t_l\}} [\frac{1}{2} * (t_h - rk_h)] + \frac{1}{2} * [t_l - rk_l)]$ subject to:

1. $\frac{1}{2} * \left[2\sqrt{k_h} - t_h\right] + \frac{1}{2} * \left[\sqrt{k_h} - t_h\right]$
$\geq \frac{1}{2} * \left[2\sqrt{k_l} - t_l\right] + \frac{1}{2} * \left[\sqrt{k_l} - t_l\right]$

2. $\frac{1}{4} * \left[2\sqrt{k_l} - t_l\right] + \frac{3}{4} * \left[\sqrt{k_l} - t_l\right]$
$\geq \frac{1}{4} * \left[2\sqrt{k_h} - t_h\right] + \frac{3}{4} * \left[\sqrt{k_h} - t_h\right]$

3. $\frac{1}{2} * \left[2\sqrt{k_h} - t_h\right] + \frac{1}{2} * \left[\sqrt{k_h} - t_h\right] \geq 0$

4.$\frac{1}{4} * \left[2\sqrt{k_l} - t_l\right] + \frac{3}{4} * \left[\sqrt{k_l} - t_l\right] \geq 0$. Simplify these into:
1. $3\sqrt{k_h} - 2t_h \geq 3\sqrt{k_l} - 2t_l$
2. $\frac{5}{4}\sqrt{k_l} - t_l \geq \frac{5}{4}\sqrt{k_h} - t_h$
3.$3\sqrt{k_h} - 2t_h \geq 0$
4.$5\sqrt{k_l} - 4t_l \geq 0$. Make 1. 4 bind. $\Longrightarrow t_l = \frac{5}{4}\sqrt{k_l}$ and $t_h = \frac{3}{2}\sqrt{k_h} - \frac{3}{2}\sqrt{k_l} + \frac{5}{4}\sqrt{k_l} = \frac{3}{2}\sqrt{k_h} - \frac{1}{4}\sqrt{k_l}$.
Solution: $k_h = \frac{9}{16r^2}, k_l = \frac{1}{4r^2}, t_l = \frac{5}{8} * \frac{1}{r}, t_h = \frac{1}{r}$.

Justify your answers whenever possible.

1. Consider an economy with $I \geq 2$ consumers, $J \geq 2$ firms and $L \geq 2$ goods. Produced goods $1, \ldots, L-1$ are produced using good L as the only input. Firm $j = 1, \ldots, J$ produces the vector $y_j \geq 0$ of outputs of the produced goods using $c_j(y_j) \geq 0$ units of good L, where c_j is strictly convex and C^2. Consumer i has an initial endowment of $e_i > 0$ units of good L, receives the share $\theta_{ij} \geq 0$ of the profit of firm j and gets utility $\phi_i(x_i) + m_i$ from consuming the vector $x_i \geq 0$ of produced goods and $m_i \in$ units of good L. The function ϕ_i is C^2, nondecreasing and concave.

a. Give a possible economic interpretation for m_i when it is negative.

The consumer owes $-m_i$ units of good L which are to be given up at a date outside the period represented in the model.

b. Interpret the convexity of the function c_j and explain its economic significance.

The strict convexity means that there are decreasing returns in every direction in production by firm j. If two production plans are possible for the firm, then using slightly less than the average of the input amounts in the two plans, the firm can simultaneously produce the average of each of the outputs from the two plans.

c. Use the notation above to give a formal definition of the set of feasible allocations in this economy.

$((x_i, m_i), (y_j))$ with $x_i \geq 0$, $y_j \geq 0$, $\sum x_i = \sum y_j$, $\sum m_i + \sum c_j(y_j) = \sum e_i$.

d. Use the notation above to give a formal definition of the set of Pareto efficient allocations in this economy.

$((\bar{x}_i, \bar{m}_i), (\bar{y}_j))$ is Pareto efficient if it is feasible and there is no feasible $((x_i, m_i), (y_j))$ such that $m_i + \phi_i(x_i) \geq \bar{m}_i + \phi_i(x_i), \forall i$, with strict inequality for some i.

e. Show that every Pareto efficient allocation maximizes a weighted sum of the consumers' utilities over the set of feasible allocations. What can be said about the weights?

The Kuhn-Tucker theorem implies that Pareto efficient allocations maximize the sum of the consumers' utilities over the set of feasible allocations. The weights are equal to each other. There is also a direct proof that does not rely on differentiability of ϕ_i and c_j.

f. Give a set of equations and/or inequalities that characterize the set of Pareto efficient allocations in this economy. Interpret the equations and/or inequalities. How many Pareto efficient allocations are there? Describe how the different Pareto efficient allocations differ from each other.

There exist $\lambda_\ell \geq 0$ such that the feasibility conditions are satisfied and $\partial_\ell \phi_i(x_i) \leq \lambda_\ell$, with equality if $x_{\ell i} > 0, \forall i$, and $\lambda_\ell \leq \partial_\ell c_j(y_j)$, with equality if $y_{\ell j} > 0, \forall j$. The Lagrange multiplier λ_ℓ can be interpreted as a price of output ℓ in a price equilibrium with transfers. The inequalities say that λ_ℓ equals the marginal utility of good ℓ for each consumer who consumes it and equals the marginal cost of good ℓ (in terms of good L) for each firm that produces ℓ.

There are infinitely many Pareto efficient allocations. They differ only in the allocation of the m_i's.

g. Is every Pareto efficient allocation part of a price equilibrium with transfers in this economy? In other words, given a Pareto efficient allocation $a = ((x_i, m_i)_i, (y_j)_j)$ does there exist a reallocation of the endowments and profit shares such that for the new initial ownership there is a competitive equilibrium with allocation a?

Yes. The first order conditions in part f are sufficient for each firm to be maximizing its profit and each consumer to be optimizing given appropriate wealth.

h. Explain how the answers to questions e and g change (if at all) under the assumption that each m_i is restricted to be nonnegative in every feasible allocation.

In e, the weights in the weighted sum of consumers' utilities are no longer necessarily equal to each other. The Pareto efficient allocations in which all the consumers get positive amounts of good L all have the same allocation of produced goods, but allocations in which some consumer gets none of good L may have different allocation of the produced goods.

2. A firm E considers entering a new market in which a single firm M is already producing. If firm E does not enter, then both firms get 2 units of profit. If firm E enters, it can do so with a big or small investment (b or s). Firm M observes whether E has entered, but not the size of E's investment. If E enters, firm M must decide whether to choose r (reduce its output price) or n (not reduce the price). The firms' profits following entry by E are in the table below. Both firms are rational expected profit maximizers and the interaction described above is common knowledge.

Player M

Player E		n	r
	b	5, 1	0, 0
	s	0, 0	1, 3

a. Draw a game tree that can represent the entire interaction between the firms starting when E considers entering the market.

b. How many pure strategies does E have in the game represented by the tree you drew in part a? Give an example of a pure strategy for E.

c. How many pure strategies does M have in the game? Give an example of one of them.

d. Which profits in the table above might seem a little strange? Why might they seem strange? Why might firms in real life make profits like those in the table?

e. Find every pure strategy Nash equilibrium (NE) of the game. Interpret the outcomes.

f. Find every pure strategy weak perfect Bayesian equilibrium (WPBE) of the game.

g. Is every pure strategy WPBE a sequential equilibrium (SE)? Is there any SE in which some player does not play a pure strategy?

h. Is there a SE with implausible beliefs? If the answer is yes, explain which SE outcomes are more plausible and which are less plausible, explain why implausible beliefs are sustainable in SE and how they might be ruled out in a reasonable, but more restrictive, solution concept. If the answer is no, explain why implausible beliefs are ruled out in SE of this game.

4. Consider a profit maximizing farmer who owns 100 acres of land (T) and must decide how much to devote to growing corn (c) and how much to wheat (w). Suppose the outputs of corn and wheat are given by the concave production functions $c = f(T_c, s_c)$ and $w = g(T_w, s_w)$, respectively, where T_i denotes the amount of land devoted to i and s_i is the amount of seed, for $i = c, w$. Let p_c and p_w denote the output prices of corn and wheat, and r_c and r_w denote the input prices of corn seed and wheat seed, respectively. Land is not traded.

a. Set up the farmer's decision problem assuming he knows all of the above information. Characterize a solution in terms of the appropriate first order conditions and interpret the conditions.

Answ.

$\max_{T_c,T_w,s_c,s_w,w,c} p_w w + p_c c - r_c s_c - r_w s_w$ s.t.$c = f(T_c, s_c)$, $w = g(T_w, s_w)$ and $T_c + T_w = 100$

or

$\max_{T_w,s_c,s_w} p_w g(T_w, s_w) + p_c f(100 - T_w, s_c) - r_c s_c - r_w s_w$

FOCs:

(T_w) $p_w g_{T_w} = p_c f_{T_c}$ equate marginal revenue products of land

(s_w) $p_w g_{s_w} = r_w$ equate marginal revenue product of s_w to factor price

(s_c) $p_c f_{s_c} = r_c$ equate value of marginal product of s_c to factor price

b. Next, assume the farmer faces two sources of uncertainty: the weather and the selling price of corn – assume the price of seed is known at the start of the season and the price of wheat is fixed. Let $\theta \in \{0, 1\}$ indicate the state of the weather, where 0 denotes "bad" and 1 denotes "good," and suppose these occur with probabilities π_0 and $\pi_1 = 1 - \pi_0$, respectively. Also assume p_c can take on two values, $p_c^h > p_c^\ell$, with probabilities π_c^h and $\pi_c^\ell = 1 - \pi_c^h$. In this case, output of each crop is stochastic and is given by $c = f(T_c, s_c; \theta)$ and $w = g(T_w, s_w; \theta)$. Assuming the random variables p_c and θ are independent, characterize a solution to the farmer's choice problem.

Answ.

Expected profit $= \pi_1 \pi_c^h p_c^h f(100-T_w, s_c; 1) + \pi_1 \pi_c^\ell p_c^\ell f(100-T_w, s_c; 1) + \pi_0 \pi_c^h p_c^h f(100-T_w, s_c; 0) + \pi_0 \pi_c^\ell p_c^\ell f(100-T_w, s_c; 0) + \pi_1 p_w g(T_w, s_w; 1) + \pi_0 p_w g(T_w, s_w; 0) - r_c s_c - r_w s_w$

Maximizing in T_w, s_c and s_w yields the conditions:

(T_w) $p_w E(g_{T_w}) = E(p_c) E(f_{T_c})$

(s_w) $p_w E(g_{s_w}) = r_w$

(s_c) $E(p_c) E(f_{s_c}) = r_c$

c. Suppose now that corn production is less susceptible to fluctuations in the weather. Indeed, consider the extreme case in which it is entirely independent of θ and achieves the value $c = f(T_c, s_c) = E_\theta f(T_c, s_c; \theta)$ (i.e., the value of the output in the certainty case is the expected value of the random output in the uncertainty case). Relative to part b, will more land necessarily be devoted to corn production in this case? If so, prove it. Otherwise, describe conditions under which the amount of land devoted to corn will increase.

Answ.

new FOC for T_w: $p_w E(g_{T_w}) = E(p_c) f_{T_c}(100 - T_w, s_c)$

vs. in part b: $p_w E(g_{T_w}) = E(p_c)[\pi_1 f_{T_c}(100 - T_w, s_c; 1) + \pi_0 f_{T_c}(100 - T_w, s_c; 0)]$

If f_{T_c} is concave, then RHS "new" is greater than the RHS "old". Hence, T_w must decrease and T_c increase.

d. Returning to the case in which both crops are stochastic as in part b, suppose instead that p_c and θ are (perfectly) correlated: when the weather is good, the supply of corn is greater and hence the market price is lower. How would this affect your answer to part b?

Answ.

Now

$P(\pi_1 \cap \pi_c^h) = 0, P(\pi_1 \cap \pi_c^\ell) = \pi_1, P(\pi_0 \cap \pi_c^h) = \pi_0, P(\pi_0 \cap \pi_c^\ell) = 0$

Hence, Expected profit =

$\pi_1[p_c^\ell f(100 - T_w, s_c; 1) + p_w g(T_w, s_w; 1)] + \pi_0[p_c^h f(100 - T_w, s_c; 0) + p_w g(T_w, s_w; 0)] - r_c s_c - r_w s_w$

Maximizing in T_w, s_c and s_w yields the conditions:

$(T_w)\ \ p_w E(g_{T_w}) = \pi_1 p_c^\ell f_{T_c}(100 - T_w, s_c; 1) + \pi_0 p_c^h f_{T_c}(100 - T_w, s_c; 0)$

$(s_w)\ \ p_w E(g_{s_w}) = r_w$

$(s_c)\ \ \pi_1 p_c^\ell f_{s_c}(100 - T_w, s_c; 1) + \pi_0 p_c^h f_{s_c}(100 - T_w, s_c; 0) = r_c$

e. In order to encourage the production of corn for use in ethanol, the government is considering two alternative policies: (1) subsidize the price of corn at the rate of σ per unit, or (2) provide crop insurance for the effect of bad weather on corn production. Assuming once again that p_c and θ are independent, discuss how both policies would work and how they would affect the farmer's decision. Can you compare the effectiveness of the two policies?

Answ.

(1) in expected profit expression in part b, replace, p_c^i with $(p_c^i + \sigma)$ throughout.

(2) Let ρ denote price per unit of insurance. Then the net expected benefit/profit of coverage x would be $-\rho x + \pi_0 x$. Hence, if $\pi_0 > \rho$, then the farmer will maximally insure. If $\pi_0 < \rho$, the optimal coverage is 0, and otherwise it is indifferent between any amounts.

f. Suppose it is possible for the farmer to "pre-sell" his corn, that is, to commit beforehand to sell his entire output of at a predetermined price. Characterize the lowest price the farmer would be willing to accept in exchange for the buyer to bear the risk associated with the price fluctuation.

Answ.

Solve

$\Pi(p) = \max_{T_w, s_c, s_w} \pi_1[pf(100 - T_w, s_c; 1) + p_w g(T_w, s_w; 1)] + \pi_0[pf(100 - T_w, s_c; 0) + p_w g(T_w, s_w; 0)] - r_c s_c - r_w s_w$

Then find p that equates $\Pi(p)$ to maximal expected profits in part b.