

# Microeconomics

Instructions: Answer any three of the following four questions. Whenever possible, justify your answers. Write your answer to each question in a **separate** bluebook. Write the number of the question on the cover of the bluebook. DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam is 4 hours.

1. A bank has two types of borrowers: the highly productive (type  $h$ ) and less productive (type  $l$ ) firms. It is known that they are equally likely. The firms finance their investments entirely through a bank loan. There is only one bank the firms can borrow from. If a type  $h$  firm gets a loan of size  $k$ , it will generate a gross profit (after subtracting  $k$ ) of  $2\sqrt{k}$ , whereas a type  $l$  firm will generate a gross profit of  $\sqrt{k}$ . The bank has an alternative investment which would yield  $rk$ , where  $r > 0$ , for investment of size  $k$ . If the bank loans  $k$  and the repayment is  $k + t$ , the bank's 'profit' is  $t - rk$ . If a type  $h$  firm (resp. type  $l$  firm), borrows  $k$  and repays  $k + t$ , it will have net profit  $2\sqrt{k} - t$  (resp.  $\sqrt{k} - t$ ). The bank maximizes its expected profit without being able to observe the type of a particular borrower.

- Set up the optimal contract problem of the bank
- Explain why the problem you formulated in part a is the right problem to solve.
- Show that at the optimum, the incentive constraint of the type  $h$  and the participation constraint of type  $l$  bind.
- Solve the optimal contract problem and interpret it.
- Now, suppose that the outcome of investment is random. A type  $h$  firm if it invests  $k$  has  $\frac{1}{2}$  probability of getting a gross profit of  $2\sqrt{k}$  and  $\frac{1}{2}$  probability of getting a gross profit of  $\sqrt{k}$ . A type  $l$  firm if it invests  $k$  has  $\frac{1}{4}$  probability of getting a gross profit of  $2\sqrt{k}$  and  $\frac{3}{4}$  probability of getting a gross profit of  $\sqrt{k}$ . Assume that repayment of  $k + t$  is not contingent on the profit outcome. Assume that the firms have unlimited liability: the firms can and must pay contracted  $t$  (as well as  $k$ ) no matter the size of the profits. The firms as well as the bank are risk neutral. Set up the problem, solve it and compare it to the solution in part d.

2. Consider an economy with  $L \geq 2$  goods,  $I \geq 2$  consumers,  $J \geq 2$  firms, and no free disposal. Produced goods  $1, \dots, L - 1$  are produced using good  $L$  as the only input. Firm  $j = 1, \dots, J$  produces the vector  $y_j \geq 0$  of outputs of the produced goods using  $c_j(y_j) \geq 0$  units of good  $L$ , where  $c_j$  is strictly convex and  $\mathcal{C}^2$ . Consumer  $i$  has an initial endowment of  $e_i > 0$  units of good  $L$ , receives the share  $\theta_{ij} \geq 0$  of the profit of firm  $j$  ( $\sum_i \theta_{ij} = 1$ , for all  $j$ ) and gets utility  $\phi_i(x_i) + m_i$  from consuming the vector  $x_i \geq 0$  of produced goods and  $m_i \in$  units of good  $L$ . The function  $\phi_i$  is  $\mathcal{C}^2$ , nondecreasing and concave.

- Give a possible economic interpretation for  $m_i$  when it is negative.
- Interpret the convexity of the function  $c_j$  and explain its economic significance.
- Use the notation above to give a formal definition of the set of feasible allocations in this economy.
- Use the notation above to give a formal definition of the set of Pareto efficient allocations in this economy.

- e. Show that every Pareto efficient allocation maximizes a weighted sum of the consumers' utilities over the set of feasible allocations. What can be said about the weights?
- f. Give a set of equations and/or inequalities that characterize the set of Pareto efficient allocations in this economy. Interpret the equations and/or inequalities. How many Pareto efficient allocations are there? Describe how the different Pareto efficient allocations differ from each other.
- g. Is every Pareto efficient allocation part of a price equilibrium with transfers in this economy? In other words, given a Pareto efficient allocation  $a = ((x_i, m_i)_i, (y_j)_j)$ , does there exist a reallocation of the endowments and profit shares such that for the new initial ownership there is a competitive equilibrium with allocation  $a$ ?
- h. Explain how the answers to questions e and g change (if at all) under the assumption that each  $m_i$  is restricted to be nonnegative in every feasible allocation.

3. A firm E considers entering a new market in which a single firm M is already producing. If firm E does not enter, then both firms get 2 units of profit. If firm E enters, it can do so with a big or small investment ( $b$  or  $s$ ). Firm M observes whether E has entered, but not the size of E's investment. If E enters, firm M must decide whether to choose  $r$  (reduce its output price) or  $n$  (not reduce the price). The firms' profits following entry by E are given in the table below. Both firms are rational expected profit maximizers and the interaction described above is common knowledge.

		Player M	
		$n$	$r$
Player E	$b$	5, 1	0, 0
	$s$	0, 0	1, 3

- a. Draw a game tree that can represent the entire interaction between the firms starting when E considers entering the market.
- b. How many pure strategies does E have in the game represented by the tree you drew in part a? Give an example of a pure strategy for E.
- c. How many pure strategies does M have in the game? Give an example of one of them.
- d. Which profits in the above table might seem a bit strange? Explain why. Why might firms in real life make profits like those in the table?
- e. Find every pure strategy Nash equilibrium (NE) of the game. Interpret the outcomes.
- f. Find every pure strategy weak perfect Bayesian equilibrium (WPBE) of the game.
- g. Is every pure strategy WPBE a sequential equilibrium (SE)? Is there any SE in which some player does not play a pure strategy?
- h. Is there a SE with implausible beliefs? If the answer is yes, explain which SE outcomes are more plausible and which are less plausible, explain why implausible beliefs are sustainable in SE and how they might be ruled out in a reasonable, but more restrictive, solution concept. If the answer is no, explain why implausible beliefs are ruled out in SE of this game.

4. Consider a profit maximizing farmer who owns 100 acres of land ( $T$ ) and must decide how much to devote to growing corn ( $c$ ) and how much to wheat ( $w$ ). Suppose the outputs of corn and wheat are given by the concave production functions  $c = f(T_c, s_c)$  and  $w = g(T_w, s_w)$ , respectively, where  $T_i$  denotes the amount of land devoted to  $i$  and  $s_i$  is the amount of seed,

for  $i = c, w$ . Let  $p_c$  and  $p_w$  denote the output prices of corn and wheat, and  $r_c$  and  $r_w$  denote the input prices of corn seed and wheat seed, respectively. Land is not traded.

- a. Set up the farmer's decision problem assuming he knows all of the above information. Characterize a solution in terms of the appropriate first order conditions and interpret the conditions.
- b. Next, assume the farmer faces two sources of uncertainty: the weather and the selling price of corn – assume the price of seed is known at the start of the season and the price of wheat is fixed. Let  $\theta \in \{0, 1\}$  indicate the state of the weather, where 0 denotes “bad” and 1 denotes “good,” and suppose these occur with probabilities  $\pi_0$  and  $\pi_1 = 1 - \pi_0$ , respectively. Also assume  $p_c$  can take on two values,  $p_c^h > p_c^\ell$ , with probabilities  $\pi_c^h$  and  $\pi_c^\ell = 1 - \pi_c^h$ . In this case, output of each crop is stochastic and is given by  $c = f(T_c, s_c; \theta)$  and  $w = g(T_w, s_w; \theta)$ . Assuming the random variables  $p_c$  and  $\theta$  are independent, characterize a solution to the farmer's choice problem.
- c. Suppose now that corn production is less susceptible to fluctuations in the weather. Indeed, consider the extreme case in which it is entirely independent of  $\theta$  and achieves the value  $c = f(T_c, s_c) = E_\theta f(T_c, s_c; \theta)$  (i.e., the value of the output in the certainty case is the expected value of the random output in the uncertainty case). Relative to part b, will more land necessarily be devoted to corn production in this case? If so, prove it. Otherwise, describe conditions under which the amount of land devoted to corn will increase.
- d. Returning to the case in which both crops are stochastic as in part b, suppose instead that  $p_c$  and  $\theta$  are perfectly correlated: when the weather is good, the supply of corn is greater and hence the market price is lower. How would this affect your answer to part b?
- e. In order to encourage the production of corn for use in ethanol, the government is considering two alternative policies: (1) subsidize the price of corn at the rate of  $\sigma$  per unit, or (2) provide crop insurance for the effect of bad weather on corn production. Assuming once again that  $p_c$  and  $\theta$  are independent, discuss how both policies would work and how they would affect the farmer's decision. Can you compare the effectiveness of the two policies?
- f. Suppose it is possible for the farmer to “pre-sell” his corn, that is, to commit beforehand to sell his entire output at a predetermined price. Characterize the lowest price the farmer would be willing to accept in exchange for the buyer to bear the risk associated with the price fluctuation.