Microeconomics

Instructions. Answer any 3 of the following 4 questions. Show all of your work and justify your answers whenever possible. Write your answer to each question in a separate bluebook. On the cover of the bluebook, write the number of the question under "Section." DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. The exam lasts 4 hours.

1. An auctioneer auctions off one indivisible object to two bidders. The bidders' (private) valuations $\{v_1, v_2\}$ are drawn independently from $[0,1]$ according to the uniform distribution. Thus, the probability of a bidder's valuation to come from $[0,x]$, $x \leq 1$ is x. A bidder knows his valuation when the auction starts but does not know the other bidder's valuation except for its distribution. This is common knowledge. We consider the first-price sealed-bid auction where the bidder with highest bid wins the object and pays his bid. When there is a tie, the winner is chosen randomly with equal probability. If a bidder does not win, he pays nothing. Assume that the bidder i maximizes, by choosing $\widehat{b}_i$, the expected surplus: $(v_i - \widehat{b}_i)\times$ the probability of winning with the bid of $\widehat{b}_i$

   (a) Suppose bidder 2 employs a strictly increasing strategy $b : [0,1] \to R_+$. If bidder 1 bids $\widehat{b}_1$ which is equal to less than $b(1)$, what is bidder 1's probability of winning?

   (b) Let $\pi\left(v_1, \widehat{b}_1\right)$ be the expected surplus of bidder 1 when his valuation is v_1 and his bid is $\widehat{b}_1$ when bidder 2 uses bid function b. Let $b_1(v_1) = \arg\max_{\widehat{b}_1} \pi\left(v_1, \widehat{b}_1\right)$ and $\Pi(v_1) = \pi(v_1, b_1(v_1))$. Assuming differentiability, derive $\frac{d}{dv_1}\Pi(v_1)$.

   (c) Now, consider the case of symmetric equilibrium where $b_1 = b$. What is the expression for $\frac{d}{dv_1}\Pi(v_1)$ in this case? From this, derive $\Pi(v_1)$ and the symmetric bid function b.

   (d) Show the pair of symmetric bidding function (b, b) derived in c. is in fact a Bayesian Nash equilibrium.

   (e) Describe the incentive compatible direct mechanism that implements the social choice function of the first price sealed bid auction described in the above.

   (f) Now, suppose the auctioneer tries the second price sealed bid auction. In the second price sealed bid auction, the highest bidder wins the object but pays the second highest bid. If the bids are equal, the winner is chosen randomly with equal probability and the winner pays the bid. The one who does not win pays nothing. Show that bidding one's valuation is a weakly dominant strategy.

   (g) Suppose that the valuation of bidder 1 is v_1. Compute the expected surplus of bidder 1 in the second price sealed bid auction and compare it with the first price sealed bid auction case computed in c. (economic environments do not change)

2. Consider an economy with two consumers 1 and 2 and one producer. There are two goods, leisure and rice. We denote a typical *consumption* allocation by $((x_1, y_1), (x_2, y_2))$, where x_1 (resp. y_1) denotes of the amount of leisure (resp. rice) consumed by consumer 1 and similarly for consumer 2. Consumer 1 has the endowment of one unit of leisure and zero rice. Consumer 2 has the endowment of one unit of leisure and 1 unit of rice. The consumers have the same utility function, $u(x_i, y_i) = x_i^{\frac{1}{2}} * y_i^{\frac{1}{2}}, i = 1, 2$. The producer has a fixed ratio, constant returns to scale production function $r = 10\min\{l, s\}$, where r stands for rice output, s stands for the rice used as inputs and l stands for the total labor employed (thus, $l = 1 - x_1 + 1 - x_2$). We normalize prices by setting the price of leisure to 1. Thus, the price of rice, p, is in terms of leisure. Assume that by its nature, a consumer cannot consume more leisure than his endowment of 1.

   (a) What are the demand functions for consumer 1 and consumer 2 as a function of the price of rice? Ignore the constraints that x_1, x_2 cannot exceed 1 for this question.

   (b) What is the profit maximizing input and output vector at different values of $p \geq 0$?

   (c) What is the competitive equilibrium allocation? What is the total supply of rice (net rice output plus rice endowment)?

(d) Now, suppose the production function is given by $r = \frac{3}{2}\min\{l, s\}$ What value of $p > 0$ would be compatible with positive output of a profit maximizing producer? At this price, what is the utility maximizing demand for consumer 2 when his demand for leisure cannot exceed 1? Either argue algebraically or draw a graph to justify your answer.

(e) What is the competitive equilibrium allocation in the case of (d)? (Imagine that the producer can borrow rice for seed from consumer 2 at zero interest rate).

(f) Is the equilibrium in (e) Pareto optimal?

3. On a given night, $N \geq 2$ consumers each attach a value of \$5 to seeing a movie at the town theater. In addition, the consumers each attach a value of $5q - (q^2/2)$ to consuming q boxes of popcorn at the theater, where q can be any nonnegative number. Popcorn can only be bought at the theater. No other consumers attach positive value to seeing the movie or to eating popcorn. The theater must pay a fixed cost \$$K$ if it admits one or more consumers and a cost of \$$q$ for q units of popcorn it sells. It has no other costs. The theater charges a two-part tariff consisting of m for admission and p per box of popcorn. It sells only to admitted consumers, and m and p can be any real numbers. The theater announces its prices and consumers choose whether to pay for admission and how much popcorn to buy. The payoff to the theater is its expected profit from its interaction with the N consumers. Consumers' payoffs are the value they attach to what they get minus what they pay. Consumers who do not see the movie get 0 payoff. All the above information is common knowledge.

(a) The interaction above determines a game played by the N consumers and the theater. Give a formal definition of a pure strategy for a consumer in this game and give an example of such a pure strategy.

(b) Find a pure strategy subgame perfect Nash equilibrium of the game when $K = 1$. Find the corresponding payoffs. What can be said about the efficiency of the outcome? Explain clearly what notion of efficiency you refer to.

(c) How do the subgame perfect Nash equilibrium outcomes vary depending on K when $K > 1$? Be as specific as possible.

(d) Could the theater raise its payoff above what it gets in part b by selling popcorn to consumers who do not pay to see the movie?

From now on, suppose that $K = 1$ and that a fraction a of the N consumers are as in the description above, but the fraction $1 - a$ attach value \$3 to seeing a movie at the theater and attach no value to popcorn, where $0 < a < 1$. Consumers know their valuations, but the theater owner and workers do not know what type any given consumer is. Everything else is as described above, and all this information is common knowledge.

(e) Find an optimal two-part tariff (m, p) for the theater and find the resulting allocation. Assume that consumers see the movie whenever they are indifferent between seeing it and not seeing it. How do the prices and the quantity of popcorn sold depend on a? What can be said about the efficiency of the outcome?

(f) Is it possible that an alternative (possibly more complex) contract could give the theater a higher payoff than what it gets in part e? If so, find and interpret such a contract. If not, show why not. You can get half of the credit for this part of the problem by setting up an appropriate optimization problem for the theater and explaining why it is appropriate and how it can be used to answer the question.

(g) Movie theaters typically charge prices well above marginal cost for popcorn, but charge nothing for access to bathrooms. Does the model above suggest any reason for this?

4. Suppose there are three agents each of whom has preferences defined over a single consumption good, x, and (numeraire) money, m. Specifically, their preferences are represented respectively by $u_1(x_1, m_1) = x_1^{\frac{1}{2}} m_1^{\frac{1}{2}}$, $u_2(x_2, m_2) = 2x_2^{\frac{1}{2}} + m_2^{\frac{1}{2}}$, and $u_3(x_3, m_3) = x_3^{\frac{1}{2}} + 2m_3^{\frac{1}{2}}$. An egalitarian social planner has a fixed amount of money, $\bar{M}$, to distribute among the three. Let M_i denote the amount allocated to agent i. The planner's (Bergson-Samuelson) social welfare function is given by $W(u_1, u_2, u_3) = \min\{u_1, u_2, u_3\}$.

(a) Explain why this might be interpreted as an egalitarian social welfare function.

(b) Assuming the agents can purchase any quantity of x at the price p, write the budget constraint facing agent i contingent on receiving M_i.

For the remainder of the problem assume $p = 1$.

(c) Assuming the social planner knows the utility functions of the agents, find the optimal allocation of $\bar{M} = 120$, denoted $\mathbf{M}^* = (M_1^*, M_2^*, M_3^*)$. Discuss the distributive implications of this outcome. Is it "fair"? Why or why not?

(d) Reevaluate part c for the case in which $\bar{M} = 60$. In this case, is the outcome "fair"? Explain.

For the remainder of the problem again assume that $\bar{M} = 120$.

(e) Argue that $\mathbf{M}^*$ is the least expensive way to achieve the optimal social welfare level, i.e., that every other way would require more than $\bar{M}$.

(f) Relative to the optimal level, evaluate the social welfare loss that would result if $\bar{M}$ were divided equally.

(g) Rather than simply allocating the aggregate quantity $\bar{M}$, suppose the agents were initially endowed with the quantities $M_1^\circ = 35$, $M_2^\circ = 35$ and $M_3^\circ = 50$, respectively. Describe a system of lump sum redistributive taxes $(T_1, T_2, T_3) \in \mathbb{R}^3$ (i.e., $T_1 + T_2 + T_3 = 0$) that, if levied on the three agents, would achieve the social optimum.

(h) Suppose the planner were required to tax only on the basis of initial income, i.e., it were constrained such that $T_1 = T_2$. Determine the (second-best) social optimum subject to this constraint and evaluate the welfare loss relative to the optimum in part c.

(i) Returning to the case *without* initial claims, suppose the planner would prefer that each agent spends at least $\frac{1}{2}$ of its income M_i on x_i. Toward this end, the planner decides to allocate $\bar{M}$ as follows. First, it allocates M_1^* to agent 1 and M_2^* to agent 2, as before. However, rather than allocate the entire amount M_3^* to agent 3 directly, it first allocates $\frac{M_3^*}{2}$ *units of* x in the form of a voucher or coupon and then allocates the other $\frac{M_3^*}{2}$ in the form of money. Discuss the effect of this on agent 3's utility and on social welfare. Is such *paternalism* consistent with the above specification of W?