Microeconomics

1. Consider two employees who work for the same firm. Both earn a salary of M and both consume (only) two commodities. Let x_{ik} denote agent i's consumption of commodity k. Agent 1 has preferences represented by the utility function $u_1(x_{11}, x_{12}) = \sqrt{x_{11}} + \sqrt{x_{12}}$ and agent 2's preferences are represented by $u_2(x_{21}, x_{22}) = \sqrt{x_{21}x_{22}}$. The prices of commodities 1 and 2 are p_1 and p_2.

The firm is in the process of downsizing and one of the two employees will be laid off (and earn 0) but presently it is not known which. Each faces the probability 0.5 of being laid off.

a. Derive each agent's indirect utility function.
Answer: $v_1 = \sqrt{\frac{M(p_1+p_2)}{p_1p_2}}, v_2 = \frac{M}{2\sqrt{p_1p_2}}$

b. Discuss the agents' risk postures or attitudes toward risk.
Answer: Agent 1 is risk averse, 2 is risk neutral

c. Compute their expected utilities.
Answer: $EU_1 = \frac{1}{2}\sqrt{\frac{M(p_1+p_2)}{p_1p_2}}, EU_2 = \frac{M}{4\sqrt{p_1p_2}}$

d. Suppose they were given the opportunity to self-insure against a bad draw (earning 0) by agreeing ex ante to share whatever income they receive and each obtain $\frac{M}{2}$. Would they both necessarily accept such an agreement?
Answer: 1 would prefer to risk share and 2 would be indifferent.

e. Would this risk-sharing arrangement be efficient? Discuss.
Answer: Yes. When the agents' income is the same in both states, their MRSs are equal.

f. Suppose both agents believe that under the self-insurance scheme the employee who is retained by the firm would renege on the agreement and the other agent would receive nothing. Hence, they consider contracting with a third party to enforce the agreement as follows. For a fee of ε, the "enforcer" will collect an amount T from the agent who continues to work and transfer that to the other agent. In that event, both agents would bear an equal share of the fee ε. Determine an appropriate transfer T that would equalize income ex post.
Answer: If $T = \frac{M}{2}$, both would have income $\frac{M}{2} - \frac{\varepsilon}{2}$, net of their portion of the fee.

g. What is the maximum value of ε such that both agents are willing to hire the enforcer?
Answer: If the winning agent reneges, their expected utilities are as in part c. With the "enforcer", their income would be $\frac{M-\varepsilon}{2}$ and their utilities would be $v_1 = \sqrt{\frac{(M-\varepsilon)(p_1+p_2)}{2p_1p_2}}$ and $v_2 = \frac{M-\varepsilon}{4\sqrt{p_1p_2}}$, respectively. Equating $EU_i = v_i$, 1 would pay up to $\frac{M}{2}$, but 2 would not be willing to pay any $\varepsilon > 0$.

3. There are some commodities for which one must acquire a taste, such as beer or cigarettes. That is, while initially unpleasant, they become enjoyable after consuming a sufficient quantity. To capture this phenomenon, consider an exchange economy with two agents, 1 and 2, and two commodities, x and y. Agent 1 has "standard" preferences represented by $u_1(x_1, y_1) = x_1y_1$, but agent 2's preferences are represented by

$$u_2(x_2, y_2) = \begin{cases} (x_2 - 1) + y_2 & for \quad x_2 \geq 1 \\ (1 - x_2) + y_2 & for \quad x_2 < 1 \end{cases}$$

a. Explain why 2's preferences capture this phenomenon.
Answer: u_2 is decreasing in x_2 for $x_2 < 1$ and increasing in x_2 for $x_2 \geq 1$.

b. Which, if any, of the standard assumptions do the preferences represented by u_2 fail to satisfy? Explain.

Answer: quasi-concavity, monotonicity.

c. Suppose there are 10 units of each commodity. Depict the economy in an Edgeworth Box.
Answer: See attached. (Note that O^2 is at the bottom left.)

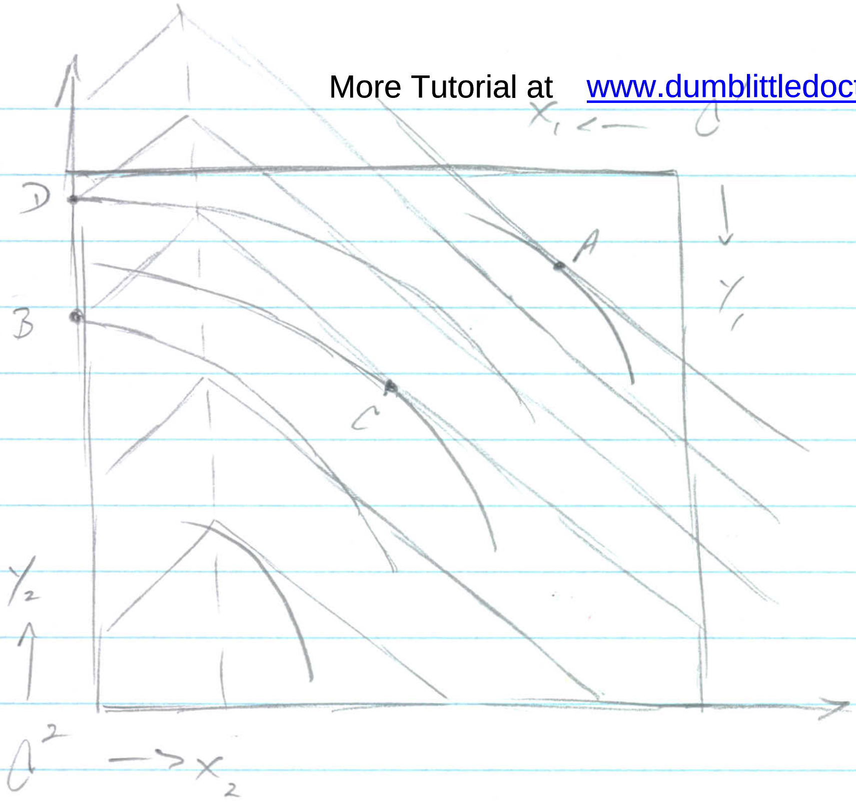
d. Identify all Pareto efficient allocations. (Hint: Do this graphically.)
Answer: There may be two types of optima: interior optima at which $MRS_1 = MRS_2 = 1$ (such as A in the accompanying figure), or boundary optima at which $x_2 = 0$ (such as B). However, not every such allocation need be efficient (neither C nor D is efficient).

e. Do the first and second classical welfare theorems apply in this case? If either one does not, does its conclusion still hold? Explain.
Answer: The second welfare theorem does not apply since preferences are not convex. The first welfare theorem applies, but it may be vacuous since a competitive equilibrium may fail to exist.

f. Suppose agent 1 initially has all 10 units of y and agent 2 has all 10 units of x. Does there exist a competitive equilibrium? Explain.
Answer: No. Given 2's preferences, the only contending price ratio would be 1. But these are not equilibrium prices.

g. Suppose instead that agent 1 initially has all the y and agent 2 has all of the x. Does there exist a competitive equilibrium in this case? Explain.
Answer: No. As in part f.

h. Finally, if agent 2's consumption set were restricted to $\{(x_2, y_2) \in \mathbb{R}^2_+ \mid x_2 \geq 1\}$, would there exist a competitive equilibrium if resources were initially allocated as in part f? Explain.
Answer: Yes. In this case, 1 would be an equilibrium price ratio.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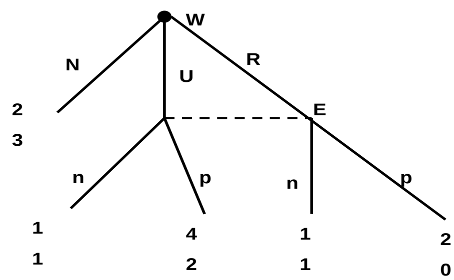


a. How many pure strategies does the worker have? Give an example of one. How many pure strategies does the employer have?

The worker has 3 pure strategies. One is to take a leave and work for a rival firm. The employer has two pure strategies.

b. Which pure strategies are rationalizable? Explain.

If the employer promotes the worker with positive probability, then the worker's expected payoff from U is higher than from R. If the employer plays n for sure, then the worker's payoff is higher with N than with R. Therefore R is never a best response and is not part of a rationalizable strategy. Every other pure strategy is rationalizable since it is a best response to some remaining strategy: n is a best response against N; p is best against U; N is best against n; U is best against p.

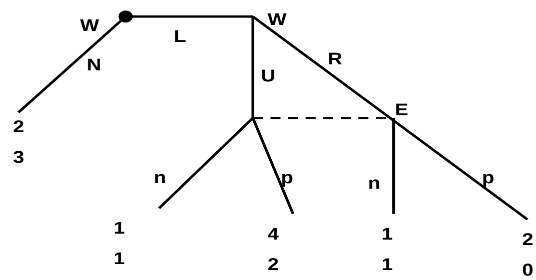
c. Find every pure strategy Nash equilibrium of the game above. Find another Nash equilibrium in mixed strategies. Show that these are Nash equilibria.

Pure strategy NE: (N, n)=(No leave, no promotion) and (U, p)=(leave for university, promotion)

Mixed strategy NE: (N, p with probability 1/3 or less). Then the worker's expected payoff from school is no more than \$2 million and the employer's expected payoff is \$3 million for all strategy choices.

d. The game tree in Figure 2 represents the same interaction between the worker and her employer described above. In this game, how many pure strategies does the worker have? Give an example of one. Does this game have a pure strategy Nash equilibrium that is **not** subgame perfect? Explain.

The game starts at the heavy black dot. L represents taking a leave. A pure strategy for the worker is a function assigning a move to every one of its information sets, so the worker has 4 pure strategies. An example is (N, U)=(no leave, university if leave taken). An example of a pure strategy NE that is not subgame perfect is ((N, U), n). Promotion, p, is a best response to U in the subgame after L.



e. Returning to the game in Figure 1, is it possible that the worker could benefit if she could commit to taking university courses if she took a leave? Explain.

Compared to a NE with no leave application, the worker does better if R is removed as a possible choice. Then when the worker applies for a leave, her only choice is U and the best response is p, so the outcome is the pure NE that the worker prefers.

f. In the game in Figure 1 (without the possibility of commitment in part e), give an argument suggesting that it is more plausible that the worker takes a leave. Is there a pure strategy sequential equilibrium in which the worker does not take a leave? Show that your answer is correct.

A leave to work for a rival is not rationalizable. The employer should not expect this strategy choice, so should promote her if she applies for a leave. Expecting this, she should apply for a leave. This is a forward induction argument. The NE with leave for university courses is also supported by risk dominance. The resistance of (N, n)=(no leave, no promotion) against (U, p)=(university, promotion) is 0. If the worker chooses N with probability less than 1 and chooses U with the remaining probability, no matter how small, then promotion is a best response. The resistance of (U, p) against (N, n) is 2/3 since university is the best response as long as n (no promotion) has probability 2/3 or less.

There are sequential equilibria in which the worker does not apply for leave. For example, consider a sequence of mixed strategies in which the probabilities of p and of U and R are $1/k$, and the probability of N is $1 - (2/k)$. For each k, hence as $k \to \infty$, the induced belief is that U and R each have probability 1/2. In that case, n a best response. When n is played, N is a best response, so the sequential equilibrium strategies are N and n with probability 1, with belief probabilities 1/2 for U and R. The same argument shows that (N, n) is also trembling hand perfect.

Answer to Problem 4

1. An employer hires a security guard to protect his residence during a month vacation he is going to take. Assume that action a available to a security guard comes from the unit interval ($a \in [0,1]$). The probability $p(a)$ of a (successful) burglary happening during a month (the contract period) is $1-a$ when an action by the guard is a. We ignore the possibility of the place burglarized more than once. The utility for the worker is $w - 10a$, where w is monthly wage in dollars. In the event of burglary, the expected loss would be \$10000. The employer knows that the worker has no other employment opportunity. We assume that both the employer and the employee are risk neutral. Let w_1 be the payment to the guard if burglary does not take place and w_0 if it does. Assume that wages need to be non-negative.

   (a) Find the conditions on w_0 and w_1 under which the optimal choice of a by the guard is 0 and conditions under which the optimal choice of a is 1. Assume that when a guard is indifferent among some actions, he will choose an action that favors the employer most.
   Answer: Worker's problem: $\max_{a\in[0,1]} (1-a)(w_0 - 10a) + a(w_1 - 10a) = (1-a)w_0 + aw_1 - 10a = w_0 + (w_1 - w_0 - 10)a$. Thus, $a = 0$ if $w_1 - w_0 < 10$, $a = 1$ if $w_1 - w_0 \geq 10$.

   (b) Find the optimal contract for the employer. What is the expected loss of the employer?
   Answer: First find an optimal contract among the contracts that induce $a = 1$. $\max_{w_0,w_1} (1-a)(-w_0 - 10000) + a(-w_1)$ subject to $w_1 - w_0 \geq 10$, $a = 1$ and $w_0 + (w_1 - w_0 - 10)a \geq 0$. This becomes: $\max_{w_0,w_1} -w_1$ subject to $w_1 - w_0 \geq 10$, $w_1 - 10 \geq 0$. So, optimal contract is $\{w_1 = 10, w_0 = 0\}$. And the expected loss -10. Next consider those contracts that induces $a = 0$. $\max_{w_0,w_1} (1-a)(-w_0 - 10000) + a(-w_1)$ subject to $w_1 - w_0 < 10$, $a = 0$ and $w_0 + (w_1 - w_0 - 10)a \geq 0$. The constraints become: $0 \leq w_1 < 10 + w_0$, $w_0 \geq 0$. Among these contracts: $-w_0 - 10000 \leq -10000$. The expected loss is -10000. Thus, optimal contract is: $\{w_1 = 10, w_0 = 0\}$ with expected loss -10.

   (c) The specification of the maximum action $a = 1$ was artificial. So, from now on, assume that $a \in [0, \infty)$ and $p(a) = \frac{1}{1+a}$. Given non-negative $\{w_0, w_1\}$, solve the optimization problem of the guard.
   Answer: $\max_{a\geq 0} \frac{1}{1+a}(w_0 - 10a) + \frac{a}{1+a}(w_1 - 10a) = \frac{w_0 + w_1 a}{1+a} - 10a = w_0 + (w_1 - w_0)\frac{a}{1+a} - 10a$. If $w_1 - w_0 \leq 0$, then the maximand decreases as a increases. So optimal a is zero. Suppose $w_1 - w_0 > 0$. The derivative of the maximand with respect to a yields: $\frac{w_1 - w_0}{(1+a)^2} - 10$ which decreases as a increases. From $\frac{w_1 - w_0}{(1+a)^2} - 10 = 0$, $a = \frac{\sqrt{w_1 - w_0}}{\sqrt{10}} - 1$. If $w_1 - w_0 \leq 10$, optimal a is zero, otherwise, it is $\frac{\sqrt{w_1 - w_0}}{\sqrt{10}} - 1$.

   (d) Utilizing the calculations in c as much as possible, formulate the problem the employer needs to solve. (you do not need to actually solve the problem).
   Answer: Principal's problem: Solve
   1. $\max_{w_0\geq 0, w_1\geq 0} \frac{1}{1+a}(-w_0 - 10000) + \frac{a}{1+a}(-w_1)$ subject to $\frac{1}{1+a}(w_0 - 10a) + \frac{a}{1+a}(w_1 - 10a) \geq 0$, $w_1 - w_0 \geq 10$, and $a = \frac{\sqrt{w_1 - w_0}}{\sqrt{10}} - 1$. Then, 2. $\max_{w_0\geq 0, w_1\geq 0} \frac{1}{1+a}(-w_0 - 10000) + \frac{a}{1+a}(-w_1)$ subject to $\frac{1}{1+a}(w_0 - 10a) + \frac{a}{1+a}(w_1 - 10a) \geq 0$, $w_1 - w_0 \leq 10$, and

$a = 0$.
Then, choose the contract that gives higher maximand.