Microeconomics

Instructions: Answer three of the following four numbered questions. Whenever possible, justify your answers. Write your answer to each question in a separate bluebook. Write the number of the question on the cover of the bluebook. DO NOT WRITE YOUR NAME OR STUDENT ID NUMBER on the bluebooks. **The exam lasts 4 hours.**

1. Consider two employees who work for the same firm. Both earn a salary of M and both consume (only) two commodities. Let x_{ik} denote agent i's consumption of commodity k. Agent 1 has preferences represented by the utility function $u_1(x_{11}, x_{12}) = \sqrt{x_{11}} + \sqrt{x_{12}}$ and agent 2's preferences are represented by $u_2(x_{21}, x_{22}) = \sqrt{x_{21}x_{22}}$. The prices of commodities 1 and 2 are p_1 and p_2.

   The firm is in the process of downsizing and one of the two employees will be laid off (and earn 0) but presently it is not known which. Each faces the probability 0.5 of being laid off.

a. Derive each agent's indirect utility function.

b. Discuss the agents' risk postures or attitudes toward risk.

c. Compute their expected utilities.

d. Suppose they were given the opportunity to self-insure against a bad draw (earning 0) by agreeing ex ante to share whatever income they receive and each obtain $\frac{M}{2}$. Would they both necessarily accept such an agreement?

e. Would this risk-sharing arrangement be efficient? Discuss.

f. Suppose both agents believe that under the self-insurance scheme the employee who is retained by the firm would renege on the agreement and the other agent would receive nothing. Hence, they consider contracting with a third party to enforce the agreement as follows. For a fee of ε, the "enforcer" will collect an amount T from the agent who continues to work and transfer that to the other agent. In that event, both agents would bear an equal share of the fee ε. Determine an appropriate transfer T that would equalize income ex post.

g. What is the maximum value of ε such that both agents are willing to hire the enforcer?

2. By contract, a worker (W) is allowed to take a leave of absence for a year, without pay. If the worker applies for the leave, she will choose either to take university courses or else work for a rival firm during the leave. The employer (E) knows this, but does not know what the worker will choose to do during the leave. If the worker applies for the leave, her employer could promote her before the leave starts in order to induce her to return afterwards. The worker and the employer care only about their (net) expected monetary returns (in present value) from the activities described. The worker's expected return is $2 million if she does not take a leave (N) and is $1 million if she takes a leave without being promoted. If she is promoted (p), her expected return is $4 million if she takes university courses (U) and $2 million if she works for a rival firm (R) during the leave.

   The employer's expected return is $3 million if the worker does not take a leave and is $1 million if the worker goes on leave without being promoted. If the worker is promoted, the employer's return is $2 million if the worker goes back to school and 0 if she works for a rival firm. All the information above is common knowledge. The interaction can be represented by the following game tree, where n represents not promoting the worker and the other letter labels are specified above. The payoffs are in millions of dollars, with the worker's payoff written above the employer's.
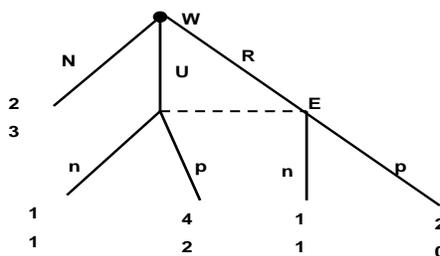
Figure 1

a. Is the strategy R weakly dominated? Is it strictly dominated? Show that your answers are correct. Hint: Consider alternative mixed strategies.
b. Which pure strategies are rationalizable? Explain.
c. Find every pure strategy Nash equilibrium of the game above. Find another Nash equilibrium in mixed strategies. Show that these are Nash equilibria.
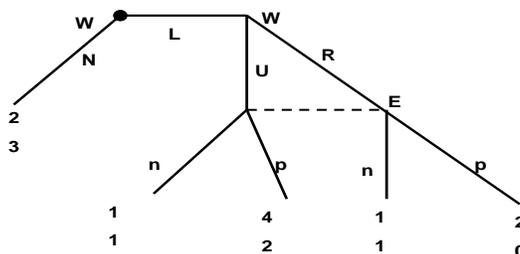

Figure 2

d. The game tree in Figure 2 represents the same interaction between the worker and her employer described above. In this game, how many pure strategies does the worker have? Give an example of one. Does this game have a pure strategy Nash equilibrium that is **not** subgame perfect? Explain.
e. Returning to the game in Figure 1, is it possible that the worker could benefit if she could commit to taking university courses in case she took a leave? Explain.
f. In the game in the Figure 1 (without the possibility of commitment in part e), give an argument suggesting that it is more plausible that the worker takes a leave than that she does not. Is there a pure strategy sequential equilibrium in which the worker does not take a leave? Show that your answer is correct.

3. There are some commodities for which one must acquire a taste, such as beer or cigarettes. That is, while initially unpleasant, they become enjoyable after consuming a sufficient quantity. To capture this phenomenon, consider an exchange economy with two agents, 1 and 2, and two commodities, x and y. Agent 1 has "standard" preferences represented by $u_1(x_1, y_1) = x_1 y_1$, but agent 2's preferences are represented by

$$u_2(x_2, y_2) = \begin{cases} (x_2 - 1) + y_2 & for \quad x_2 \geq 1 \\ (1 - x_2) + y_2 & for \quad x_2 < 1 \end{cases}$$

a. Explain why 2's preferences capture this phenomenon.

b. Which, if any, of the standard assumptions do the preferences represented by u_2 fail to satisfy? Explain.

c. Suppose there are 10 units of each commodity. Depict the economy in an Edgeworth Box.

d. Identify all Pareto efficient allocations. (Hint: Do this graphically.)

e. Do the first and second classical welfare theorems apply in this case? If either one does not, does its conclusion still hold? Explain.

f. Suppose agent 1 initially has all 10 units of y and agent 2 has all 10 units of x. Does there exist a competitive equilibrium? Explain.

g. Suppose instead that agent 1 initially has all the y and agent 2 has all of the x. Does there exist a competitive equilibrium in this case? Explain.

h. Finally, if agent 2's consumption set were restricted to $\{(x_2, y_2) \in \mathbb{R}^2_+ \mid x_2 \geq 1\}$, would there exist a competitive equilibrium if resources were initially allocated as in part f? Explain.

4. An employer hires a security guard to protect his residence during a month vacation he is going to take. Assume that action a available to a security guard comes from the unit interval ($a \in [0, 1]$). The probability $p(a)$ of a (successful) burglary happening during the month (the contract period) is $1 - a$ when an action by the guard is a. We assume that at most one burglary can occur. The utility for the guard is $w - 10a$, where w is monthly wage in dollars. In the event of burglary, the expected loss would be \$10000. The employer knows that the guard has no other employment opportunity. We assume that both the employer and the employee are risk neutral. Let w_1 be the payment to the guard if no burglary occurs and let w_0 be the payment if it does. Assume that wages need to be non-negative.

a. Find the conditions on w_0 and w_1 under which the optimal choice of a by the guard is 0 and conditions under which the optimal choice of a is 1. Assume that when a guard is indifferent among some actions, he will choose an action that favors the employer most.

b. Find the optimal contract for the employer. What is the expected loss of the employer?

c. The specification of the maximum action $a = 1$ was artificial. So, from now on, assume that $a \in [0, \infty)$ and $p(a) = \frac{1}{1+a}$. Given non-negative $\{w_0, w_1\}$, solve the optimization problem of the guard.

d. Using the calculations in c as much as possible, formulate the problem the employer needs to solve. (You do not actually need to solve the problem.)