Microeconomics

Question 1. Consider a labor market in which times may be either good or bad. If times are bad (state 1), the market wage is $x_1 = 1$. If times are good (state 2), the market wage is $x_2 = 4$. Workers in this market are expected utility maximizers, with utility function

$$U(x_1, x_2) = \pi_1 u(x_1) + \pi_2 u(x_2) = \pi_1(10x_1 - x_1^2) + \pi_2(10x_2 - x_2^2).$$

1.1 *(10 points)* Define what it means for the function $u(x_i)$ to be strictly concave. Is the function $u(x_i) = 10x_i - x_i^2$ strictly concave? Explain why or why not, as precisely as you can. Are these workers risk averse, risk neutral, or risk seeking?

1.2 *(8 points)* Now consider a firm that hires workers in this market. One thing the firm could do is pay the workers the market wage, 1 in state 1 and 4 in state 1. Assuming that the probability of state 1 is 2/3, what is the expected wage, and what is the workers' expected utility, from such an arrangement?

1.3 *(8 points)* Suppose instead the firm makes an agreement with its workers to pay them the expected wage $\bar{x}$, no matter how the market turns out. Does this make the workers better or worse off? Explain why. Formulate an equation identifying the lowest wage (call it $\underline{x}$) the firm could pay workers under such a scheme, and still leave them as well off as they are under the market-wage arrangement. You need not solve this equation.

1.4 *(4 points)* One of the "puzzles" of labor markets is that wages tend to be sticky, meaning that they do not tend to fall very much during a recession. In light of your answers to [1.1]–[1.3], what might be one explanation for this phenomenon? Answer this question very briefly.

Question 2. Consider a perfectly competitive firm that hires quantities x_1 and x_2 of inputs 1 and 2, at prices p_1 and p_2, and sells the resulting output in a market in which it receives price p. The production function is given by $f(x_1, x_2) = \ln x_1 + \ln x_2$.

2.1 *(15 points)* Formulate the profit maximization problem for this firm and find the first-order conditions. State (but do not evaluate) the second-order conditions. Find the demand function for good 1.

2.2 *(10 points)* Form the cost-minimization problem for this firm, and find the associated first-order conditions. You do not need to find the conditional demand functions. Show that marginal cost is given by the Lagrange multiplier (or its negative, depending on how you set up your constraint).

Question 3.   Ann has utility function $U(x_1, x_2) = x_1 x_2$ and income 8.

3.1 *(5 points)* Find Ann's demand for good 1.

3.2 *(8 points)* Suppose that the relaxation of pollution regulations causes the price of good 1 to fall from $p_1 = 2$ to $p_1 = 1$. Use the concept of consumer surplus to calculate the value of this price decrease to the Ann. Suppose that as a result of this regulation relaxation, increased pollution causes pollution-created costs per consumer to increase by 2. Is this regulation relaxation a good idea? Why or why not? Answer this last part briefly.

Question 4.   Consider an individual whose utility function over monetary lotteries is given by $U(x_1, x_2, \pi_1, \pi_2) = \pi_1 u(x_1) + \pi_2 u(x_2) = \frac{1}{2} x_1^{\frac{1}{2}} + \frac{1}{2} x_2^{\frac{1}{2}}$.

4.1 *(12 points)* Now suppose the person in question has incomes (w_1, w_2) in states 1 and 2, with $w_1 > w_2$. Suppose she is offered an actuarially fair insurance policy, consisting of a premium P and a level of compensation C. Formulate the relationship between P and C that must hold if the insurance policy is to be actuarially fair. Use this to formulate the utility maximization problem this person faces, identifying clearly the budget constraint. Find the first-order conditions for the maximization problem and the equilibrium consumption levels.

4.2 *(10 points)* Now suppose that instead of earning zero expected payoff, the insurance company earns an expected payoff of Δ. Formulate the consumer's utility maximization problem, paying special attention to the budget constraint, under this assumption. Explain what it means to fully insure, and without taking first-order conditions or solving, explain why you expect this person to do so.

4.3 *(10 points)* Define what it means for $U(x_1, x_2)$ to represent convex preferences over bundles (x_1, x_2). Are this person's preferences convex? Explain why or why not, as precisely as you can.