

# Microeconomics

**Question 1.** Consider a person who chooses consumption  $x_1$  and  $x_2$  in the current ( $x_1$ ) and future ( $x_2$ ) period to maximize the utility function  $U(x_1, x_2) = x_1^2 x_2$ . This person has income  $I_1 > 0$  in the first period and  $I_2 > 0$  in the second.

- 1.1 [10 points] Formulate the relevant utility-maximization problem, find the first order conditions, and find the demand curve for  $x_2$ .
- 1.2 [5 points] Let  $I_1 = I_2 = 100$  and  $r = 0.1$ . Does this person borrow or save money, or neither?
- 1.3 [5 points] Now suppose the interest rate increases. How are current and future consumption affected? What happens to the amount this person borrows or saves?

**Question 2.** Consider the utility function

$$u(x_1, x_2) = x_1 x_2^3,$$

again with accompanying budget constraint  $p_1 x_1 + p_2 x_2 = I$ .

- 2.1 [5 points] Form the Lagrange function associated with this utility maximization problem and find the first-order conditions.
- 2.2 [10 points] Find the Bordered Hessian for this constrained maximization problem. State the sufficient second-order conditions. Once you have stated these second-order conditions in the form of inequalities involving determinants, you need proceed no further. You do not have to evaluate the determinants.
- 2.3 [5 points] Define quasiconcavity. Give a graphical (not algebraic) argument that this utility function is quasiconcave.
- 2.4 [5 points] Solve for the demand function for good  $x_1$ .
- 2.5 [5 points] Find the own price elasticity of demand for good 1, and explain what this tells you about the demand for good 1.
- 2.6 [5 points] Find the indirect utility function.
- 2.7 [10 points] Find the compensated demand function for good  $x_1$ , and then verify that the Slutsky equation holds for this good.

**Question 3.** Consider a person who chooses an amount of consumption  $c$  and nonworking or leisure time  $h$  to maximize the utility function  $U(c, h)$  subject to the constraint  $c + wh = wT$ .

- 3.1 [5 points] Formulate the associated utility-maximization problem and find the first-order conditions for utility maximization. Do not worry about second-order conditions.
- 3.2 [10 points] Now suppose the an income tax is imposed on this person. Hence, the take-home wage rate is reduced to  $(1 - t)w$ , where  $t$  is the tax rate. The amount of tax collected is given by  $tw(T - h)$ , since this is the tax rate times the amount of money the person earns. Formulate the new budget constraint for the utility maximization problem and find the new first-order conditions for utility maximization.
- 3.3 [5 points] Suppose alternatively that a lump-sum tax of size  $L$  is imposed. Under this tax, the person in question must pay  $L$  in taxes, regardless of how much they earn or what they consume. Write the new budget constraint.
- 3.4 [15 points] Suppose the income-tax rate  $t$  and the lump-sum tax  $L$  are chosen so as to raise the same amount of tax revenue from this person. Identify the relationship that must hold if the two taxes are to raise the same revenue. Draw a graph with the pretax budget line and the two post-tax budget lines, and the two post-tax consumption bundles. Which tax makes the person better off, i.e., leaves the person with a higher utility? Why? Be as precise as you can.