Microeconomics

1 Question 1

1.1

The budget constraint:

$$m_1 + \frac{m_2}{1+r} = x_1 + \frac{x_2}{1+r}$$

The Langrangian:

$$L(x_1, x_2, \lambda) = x_1^{1/3} + x_2^{1/3} + \lambda\left(m_1 + \frac{m_2}{1+r} - x_1 - \frac{x_2}{1+r}\right)$$

The first-order conditions:

$$L_{x_1} = (1/3)x_1^{-2/3} - \lambda = 0$$

$$L_{x_2} = (1/3)x_2^{-2/3} - \frac{\lambda}{1+r} = 0$$

$$L_\lambda = m_1 + \frac{m_2}{1+r} - x_1 - \frac{x_2}{1+r} = 0$$

Dividing the first two first order condition by one another yields:

$$\frac{x_1^{-2/3}}{x_2^{-2/3}} = \frac{1}{1/(1+r)} = 1+r$$

$$\Rightarrow x_2 = (1+r)^{3/2}x_1$$

Plugging this expression into the budget constraint gives:

$$m_1 + \frac{m_2}{1+r} = \frac{x_2}{(1+r)^{3/2}} + \frac{x_2}{1+r} = x_2\left(\frac{1}{(1+r)^{3/2}} + \frac{1}{1+r}\right).$$

This is now a somewhat messy expression. It suffices to simply divide both sides by the coefficient on x_2. There is no economic insight to be gained from doing more algebra, so it is fine to simply do this division and stop. If you would like a somewhat neater solution, multiply both sides by (1 + r) and collect x_2 terms:

$$(1+r)m_1 + m_2 = x_2(1 + \frac{1}{\sqrt{1+r}})$$

$$x_2 = (\sqrt{1+r})\frac{((1+r)m_1 + m_2)}{(\sqrt{1+r}+1)}$$

1.2

$$\epsilon_{x_2,m_2} = \frac{dx_2}{dm_2}\frac{m_2}{x_2} = \frac{\sqrt{1+r}}{\sqrt{1+r}+1}\frac{m_2}{\sqrt{1+r}\frac{(1+r)m_1+m_2}{\sqrt{1+r}+1}} =$$

$$\frac{m_2}{(1+r)m_1+m_2}$$

Notice that second-period consumption increases as does second-period income, but the elasticity is less than one. Some of the increase in second-period income is channelled into increasing first-period consumption. In effect, people partly offset the anticipated boost in second-period income by saving less. As a result, you can increase future consumption by increasing future incomes—for example, you can increase the consumption of retired people by increasing their social security payments—but you should not expect a one-for-one increase in future consumption. Some of the increased income will show up in earlier consumption.

1.3

New budget constraint:

$$m_1 - \Delta + \frac{m_2 + (1+r)\Delta}{1+r} = x_1 + \frac{x_2}{1+r}$$

$$\Rightarrow m_1 + \frac{m_2}{1+r} = x_1 + \frac{x_2}{1+r}$$

At this point, there is no more algebra that needs to be done. Notice that the set of (x_1, x_2) combinations satisfying the budget constraint has not changed. Since preferences have not changed either, the demand function for good 2 is the same as in part 1.1. The investment-based social security system thus has no effect at all.

1.4

Neither consumption of good 2 nor utility depends on Δ. The government is transferring money across time for the consumer at the same interest rate the consumer faces. Thus, the consumer can simply borrow the money that was moved to period 2 at the same interest rate at which the transferred money was invested, thereby restoring his or her original position. We thus shouldn't be surprised that the transfer makes no difference.

In light of this, you might reasonably wonder what good social security does. The most important thing to note in this respect is that, though originally designed as an investment-based program, our social security system has long been a pay-as-you-go transfer scheme–money is taken from current contributors and paid to current recipients. This can affect the budget constraint of both. In addition, to the extent that social security does act as an investment-based program, it may be able to invest at a higher rate of return than can individuals. (Interestingly, proposals to convert part of social security to private investment accounts are motivated by the observation that social security traditionally investment in assets that did not give a particularly high rate of return.) The most difficult possibility to model is the claim that young people may not be in a position to make coherent decisions reflecting their retirement, and hence social-security-based "forced savings" can make them better off.

2 Question 2

2.1

Budget Constraint:

$$wT - wx_1 - wtx_2 = x_2$$

$$L(x_1, x_2, \lambda) = x_1x_2^3 + \lambda(wT - wx_1 - wtx_2 - x_2)$$

First Order Conditions:

$$L_1 = x_2^3 - \lambda w = 0$$

$$L_2 = 3x_1x_2^2 - \lambda wt - \lambda = 0$$

$$wT - wx_1 - wtx_2 - x_2 = 0$$

Dividing the first two FOC's gives:

$$\frac{x_2^3}{3x_1x_2^2} = \frac{w}{1+wt}$$

$$\Rightarrow x_2 = \frac{3x_1w}{1+wt}$$

Plugging into the budget constraint gives:

$$wT - wx_1 - wt(\frac{3x_1w}{1+wt}) = \frac{3x_1w}{1+wt}$$

Dividing both sides by w and collecting x_1 terms gives:

$$T = x_1(1 + \frac{3}{1+wt} + \frac{3wt}{1+wt}) = x_1(\frac{4*(1+wt)}{1+wt}) \Rightarrow x_1 = \frac{T}{4}$$

$$x_2 = (\frac{3}{4})(\frac{wT}{1+wt})$$

There are a variety of other forms for these demand curves that reflect less simplification, but are again economically equivalent and hence just as good for our purposes. If you ended this part of the question with a different budget constraint and hence different demand functions, all is not lost, as long as you proceed consistently.

2.2

$$\frac{dx_2}{dt} = -\frac{3w^2T}{4(1+wt)^2} < 0$$

So if t decreases, consumption of x_2 increases. Total amount of time spent working = $T - x_1 - tx_2$. So

$$\frac{d(Work)}{dt} = -\frac{dx_1}{dt} - x_2 - t\frac{dx_2}{dt}$$

Plugging in, we see:

$$\frac{d(Work)}{dt} = 0 - (\frac{3}{4})(\frac{wT}{1+wt}) + t(\frac{3w^2T}{4(1+wt)^2}) = \frac{-3wT}{4(1+wt)^2} < 0$$

So as t decreases, time spent working increases. This tells us that as the amount of time required to consume x_2 decreases, this person spends more time working. For example, one explanation for the increase in the amount of time that married women spend working is that the amount of time they spend on activities tied to consumption, such as food preparation, has decreased.

2.3

$$\frac{dx_1}{dw} = 0$$

As the wage increases, leisure time remains unchanged. This is usually described as a case in which the income and substitution effects associated with the increased wage exactly offset one another.

$$\frac{d(Work)}{dw} = -\frac{dx_1}{dw} - t\frac{dx_2}{dw}$$

$$\Rightarrow 0 - t((\frac{3}{4})(\frac{T}{1+wt}) - \frac{3wtT}{4(1+wt)^2}) = -\frac{3tT}{4(1+wt)^2} < 0$$

Hence, as the wage increases, time spent working decreases. One might have expected this to go the other way—as the wage increases, it is more lucrative to work, and so you might expect people to work more. In this case the income effect overwhelms the substitution effect, leading to less work.

3

3.1

$U(x_1, x_2)$ represents convex preferences over bundles (x_1, x_2) if, for any two bundles (x_1, x_2) and (x_1', x_2') such that $U(x_1, x_2) = U(x_1', x_2')$, we have:

$$U(\alpha x_1 + (1-\alpha)x_1', \alpha x_2 + (1-\alpha)x_2') \geq U(x_1, x_2) \ \forall \alpha \in [0,1]$$

Alice's preferences are not convex. To see this, take $(x_1, x_2) = (1, 3)$, and $(x_1', x_2') = (3, 1)$, and $\alpha = .5$. Then we have:

$$U(\alpha x_1 + (1-\alpha)x_2) = (.5)(.5*3 + .5*1)^2 + .5(.5*1 + .5*3)^2 = 4 < (.5)(1^2) + (.5)(3^2) = 5$$

This shows that these preferences violate convexity.

3.2

Alice is risk seeking if:

$$U(x_1, x_2) = \pi_1 u(x_1) + \pi_2 u(x_2) > u(\pi_1 x_1 + \pi_2 x_2)$$

for all pairs (x_1, x_2) (i.e., if the consumer prefers any risky lottery to a certain outcome with the same expected value as the risk lottery). You can square both sides of this function and verity that the inequality holds, or you can note that this is equivalent to having a convex u() function. Thus, Alice is risk seeking if $\frac{d^2u}{dx^2} > 0$.

If the inequality is reversed, Alice is risk averse. In this case, we see that

$$\frac{du}{dx} = 2x \Rightarrow \frac{d^2u}{dx^2} = 2 > 0$$

So, Alice is risk seeking.

3.3

$$\max_{x_1, x_2} \ (1/2)x_1^2 + (1/2)x_2^2 \qquad s.t. \ (1/2)x_1 + (1/2)x_2 = (1/2)w_1 + (1/2)w_2, \ x_1 \geq 0, \ x_2 \geq 0.$$

If you want to make this look more like the budget constraints with which we worked in class, replace w_1 by w and w_2 by $w - L$. The intuition behind this budget constraint is that the expected value of an difference between consumption and income in state 1 $(1/2(x_1 - w_1))$ must be balanced by an equivalently-sized by oppositely-signed difference in state 2 $(1/2(w_2 - x_2))$.

Now solving this problem is potentially difficult if you simply start taking derivatives. There is no algebra required. The key observation here is that Alice likes risk. As a result, she will use this insurance policy to amplify her risk, not reduce it. Having made this observation there are several ways that you might answer this question, all of them on the right track. First, if we allow Alice to have negative consumption in one state, this problem has no solution. She will make her utility arbitrarily large by choosing an arbitrarily large positive consumption in one state and arbitrarily "large" negative consumption in the other. A more realistic solution is to require Alice's consumption to be positive. Then, because Alice is risk seeking, she prefers to concentrate all her consumption in one state. Since both states are equally likely, and she has the same preference for money in each state, she is indifferent to the choice of state in which she will receive her consumption. Thus, the solution is either:

$$x_1 = \frac{(1/2)w_1 + (1/2)w_2}{(1/2)} = w_1 + w_2, x_2 = 0$$

or

$$x_1 = 0, x_2 = \frac{(1/2)w_1 + (1/2)w_2}{(1/2)} = w_1 + w_2$$

There might be further constraints—an insurance company might impose a constraint that an individual not buy a policy that increases the individual's risk. If this is the case, then Alice will simply consume her endowment, $(x_1, x_2) = (w_1, w_2)$. This seems the obvious answer.

3.4

Bob's utility maximization problem is

$$\max_{x_1,x_2} (1/2)\sqrt{x_1} + (1/2)\sqrt{x_2} \ s.t.\ p_1x_1 + p_2x_2 = p_1w_1 + p_2w_2$$

Once gain, this problem can be solved without taking any derivatives or doing any algebra. Since prices are the same for both goods, and Bob's preferences are symmetric for the two goods (and marginal utility is declining in the consumption of each good), he will split his income (20) equally across the two goods. So $x_1^B = 10, x_2^B = 10$. Thus, since his original endowment is $(0, 20)$, Bob's excess demand for good 1 is 10, and his excess demand for good 2 is -10. If Alice accepts the trade, she will receive utility:

$$U(0, 20) = (1/2)(0^2) + (1/2)(20^2) = 200 > U(10, 10) = (1/2)(10^2) + (1/2)(10^2) = 100$$

So Alice will accept the trade. There is no trade at these prices that she likes better. At the proposed trade, she is already maximizing the expected value of her income, and she has maximized risk by receiving all her income in one period, there is no further one-for-one trade that can make her better off.

In terms we have used in class, this set of prices gives us a competitive equilibrium. This is the case even though Alice's preferences are not convex. Convex preferences are a commonly-invoked sufficient condition for the existence of a competitive equilibrium but, as this example shows, they are not necessary.