Microeconomics

Question 1. Consider an individual who has a utility function $u(x_1, x_2) = \sqrt[3]{x_1} + \sqrt[3]{x_2}$ for consumption x_1 in the present and x_2 in the future. Think of x_i as a composite good to be interpreted as the amount of money this person spends on consumption in period i. This current-period income is m_1 and future income is m_2. The rate of interest is r.

1.1 *(10 points)* State the budget constraint for this consumer. Solve this person's utility-maximization problem to find the demand function for good 2. Ignore second order conditions.

1.2 *(5 points)* Suppose the government would like to increase consumption in period 2, and does so by increasing m_2. (For example, Medicaid is often motivated by a desire to increase the equivalent of period-2 consumption.) Find the elasticity of x_2 with respect to m_2, second period income.

1.3 *(10 points)* Now suppose the government institutes an investment-based social security program. In terms of our model, the individual's period-1 income is reduced by some amount Δ, with the sum Δ invested and $(1+r)\Delta$ returned to the individual in period 2, so that this individual's income (m_1, m_2) is changed to $(m_1 - \Delta, m_2 + (1+r)\Delta)$. Find the new demand functions for good 2 (ignoring second order conditions).

1.4 *(5 points)* How does the consumption of good 2 depend on Δ? How does the consumer's utility depend on Δ? (These answers might be surprising but should be obvious; if not, do not spend too much time here.)

Question 2. Consider a person who consumes two goods, x_1 and x_2. Good 2 is a composite consumption good, and so has price 1, but consumption of x_2 units of good two requires tx_2 units of time, so that t is the time cost per unit of good x_2. (For example, time must be spent preparing food in order to consume it.) Think of x_1 as "leisure" time that is not spent working and not spent fulfilling the time-cost of consuming x_2. The person has a total of time T available, and earns w per unit of time spent working. All of time T is consumed either in x_1, working, or fulfilling the time cost of consuming x_2. Suppose this persons utility function is $U(x_1, x_2) = x_1 x_2^3$.

2.1 *(10 points)* State the budget constraint for this consumer. Solve this person's utility-maximization problem to find the demand functions for goods x_1 and x_2. Ignore second order conditions.

2.2 *(10 points)* Suppose t decreases. For example, new technologies may decrease food preparation time. What is the effect on the consumption of good 2? What is the effect on the amount of time this person spends working? Be precise.

2.3 *(10 points)* Suppose the wage w increases. What is the effect on the amount of time this person spends working, and on the amount of leisure time (x_1) she consumes?

Question 3. Consider an individual (Alice) whose expected utility function over monetary lotteries is given by $U(x_1, x_2) = \frac{1}{2}u(x_1) + \frac{1}{2}u(x_2) = \pi_1 x_1^2 + \pi_2 x_2^2$.

3.1 *(10 points)* Define what it means for $U(x_1, x_2)$ to represent convex preferences over bundles (x_1, x_2). Are Alice's' preferences convex? Show that they are, or give a counterexample.

3.2 *(10 points)* Define what it means for Alice to be risk seeking and risk averse. Identify whether Alice is a risk seeking or risk averse, and show this is the case.

3.3 *(10 points)* Now suppose Alice has incomes (w_1, w_2) in states 1 and 2. Suppose Alice is offered an actuarially fair insurance policy. Formulate the utility maximization problem she faces, identifying clearly what she chooses and the budget constraint. What is the solution to this problem? (If you find yourself doing algebra here that is not working out, you are on the wrong track. Think about it carefully - if there is a trick question on this exam, this is it.)

3.4 *(10 points)* Now suppose there is a second individual (Bob), with utility function $U(x_1, x_2) = \frac{1}{2}u(x_1) + \frac{1}{2}u(x_2) = \frac{1}{2}\sqrt{x_1} + \frac{1}{2}\sqrt{x_2}$. Alice's income is $(10, 10)$ while Bob's is $(0, 20)$. Suppose $p_1 = p_2 = 1$. How much of goods 1 would Bob like to buy and how much good 2 would he like to sell at these prices, i.e., what are Bob's excess demands? Would Alice be willing to make this trade (i.e., sell Bob the good 1 he wants and buy the good 2 he wants to sell, at price $p_1 = p_2 = 1$)? Is there any other trade at these prices that Alice would like better?