

# Microeconomics

## 1 Question 1

1.1 Preferences (strongly) monotonic if, whenever  $x \geq x'$  (with at least one of the inequalities strict, i.e., whenever bundle  $x'$  has at least as much of every good as does  $x$ , and more of at least one good), then  $x \succ x'$  ( $x$  is preferred to  $x'$ ). Differentiating the utility function in this case gives:

$$\begin{aligned}\frac{du}{dx_2} &= 1 > 0 \\ \frac{du}{dx_1} &= 10 - 2x_1\end{aligned}$$

These derivatives are both positive (i.e., the second is positive, since the first always is) whenever  $x_1 < 5$ . Hence, we have monotonic preferences for all bundles in which  $x_1 < 5$ . This is accordingly a reasonable utility function for those cases in which people typically consume less than 5 of  $x_1$ .

1.2 The Lagrange function is:  $\mathbf{L} = 10x_1 - x_1^2 + x_2 + \lambda(m - p_1x_1 - p_2x_2)$ . The first order conditions are:

$$\begin{aligned}\frac{d\mathbf{L}}{dx_1} &= 10 - 2x_1 - \lambda p_1 = 0 \\ \frac{d\mathbf{L}}{dx_2} &= 1 - \lambda p_2 = 0 \\ \frac{d\mathbf{L}}{d\lambda} &= m - p_1x_1 - p_2x_2 = 0\end{aligned}$$

Dividing the first two equations above to get  $10 - 2x_1 = \frac{p_1}{p_2}$ , or  $x_1 = 5 - \frac{p_1}{2p_2}$  gives the demand function for good 1.

1.3 The price elasticity is:

$$\begin{aligned} -\frac{dx_1}{dp_1} \frac{p_1}{x_1} &= \frac{1}{2p_2} \frac{p_1}{\frac{10p_2 - p_1}{2p_2}} \\ &= \frac{p_1}{10p_2 - p_1} \end{aligned}$$

For any value of  $p_2$ , the good is unitary elastic when  $\frac{p_1}{10p_2 - p_1} = 1$ , or  $p_1 = 5p_2$ . It is elastic when  $p_1 > 5p_2$ , and inelastic when  $p_1 < 5p_2$ .

Taking  $p_2 = 1$ , the expression  $p_1 x_1 = p_1 \left(5 - \frac{p_1}{2}\right)$  (revenue/expenditure) hits the horizontal axis at  $p_1 = 0$  and at  $p_1 = 10$ . It is decreasing when  $x_1$  is elastic (ie,  $p_1 > 5$ ), constant (maximum) when it's unitary elastic (ie,  $p_1 = 5$ ), and increasing when it's inelastic (ie,  $p_1 < 5$ ). The more elastic the good is, the more opportunities the consumer has to substitute away from the good, and the more slowly does expenditure increase or the faster does expenditure drop (and hence the slope of total revenue declines).

## 2 Question 2

2.1 The indifference curve can be rewritten as  $(x_1 - 2)(x_2 - 3) = c$ , or  $x_2 = \frac{c}{x_1 - 2} + 3$ . It's a hyperbola with  $(2, 3)$  as the vertex. and with  $x_1 = 2$  and  $x_2 = 3$  as asymptotes. You can interpret 2 and 3 as minimum or subsistence levels of goods  $x_1$  and  $x_2$ ; utility is negative (and it become meaningless to talk about utility maximization) if the consumption of one good drops below its associated level.

2.2 The Lagrange function is:  $\mathbf{L} = (x_1 - 2)(x_2 - 3) + \lambda(m - p_1 x_1 - p_2 x_2)$ . The first order conditions are:

$$\begin{aligned} \frac{d\mathbf{L}}{dx_1} &= x_2 - 3 - \lambda p_1 = 0 \\ \frac{d\mathbf{L}}{dx_2} &= x_1 - 2 - \lambda p_2 = 0 \\ \frac{d\mathbf{L}}{d\lambda} &= m - p_1 x_1 - p_2 x_2 = 0 \end{aligned}$$

2.3 Notice that  $\mathbf{L}_{11} = \mathbf{L}_{22} = 0$ , and  $\mathbf{L}_{12} = \mathbf{L}_{21} = 1$ . The bordered Hessian is therefore:

$$\begin{bmatrix} 0 & 1 & -p_1 \\ 1 & 0 & -p_2 \\ -p_1 & -p_2 & 0 \end{bmatrix}$$

The determinant of the first minor principle is  $(0)(0) - p_1^2 < 0$ , and the determinant of the second minor principle is (using the first row):  $p_1 p_2 + p_1 p_2 > 0$ . The second order condition holds.

2.4 Dividing the first two equations above to get:

$$\begin{aligned} \frac{x_2 - 3}{x_1 - 2} &= \frac{p_1}{p_2} \\ x_2 &= 3 + (x_1 - 2) \frac{p_1}{p_2} \end{aligned}$$

Substitute into the budget constraint to get:

$$\begin{aligned} p_1 x_1 + p_2 \left( 3 + (x_1 - 2) \frac{p_1}{p_2} \right) &= m \\ x_1 &= \frac{m - 3p_2 + 2p_1}{2p_1} = \frac{m - 3p_2}{2p_1} + 1 \end{aligned}$$

Cross price elasticity and income elasticities are:

$$\begin{aligned} \frac{dx_1}{dp_2} \frac{p_2}{x_1} &= -\frac{3}{2p_1} \frac{p_2}{x_1} < 0 \\ \frac{dx_1}{m} \frac{m}{x_1} &= \frac{1}{2p_1} > 0 \end{aligned}$$

Thus the two goods are complements and consumption of  $x_1$  increases with income.

### 3 Question 3

3.1 The budget constraint becomes  $(1.5p_1)x_1 + p_2x_2 = m$ . With the tax, the first order condition becomes:

$$\begin{aligned}\frac{x_2^2}{2x_1x_2} &= \frac{1.5p_1}{p_2} \\ \Rightarrow \frac{x_2}{2x_1} &= \frac{1.5p_1}{p_2} \\ \Rightarrow x_2 &= \frac{3p_1x_1}{p_2}\end{aligned}$$

Substitute the demand for  $x_2$  into the budget constraint to get  $(1.5p_1)x_1 + p_2\frac{3p_1x_1}{p_2} = m$ . Solving yields  $x_1 = \frac{m}{4.5p_1} = 2$  and  $x_2 = \frac{2m}{3p_2} = 6$ . The amount of tax collected by the government is  $0.5p_1x_1 = 10$ . Alice's utility, in turn, is  $x_1x_2^2 = 72$ .

3.2 Now the budget constraint becomes  $p_1x_1 + (1.2p_2)x_2 = m$ . With the tax, the first order condition becomes:

$$\begin{aligned}\frac{x_2^2}{2x_1x_2} &= \frac{p_1}{1.2p_2} \\ \frac{x_2}{2x_1} &= \frac{p_1}{1.2p_2} \\ x_1 &= \frac{0.6p_2x_2}{p_1}\end{aligned}$$

Substitute the demand for  $x_1$  into the budget constraint to get  $p_1\frac{0.6p_2x_2}{p_1} + (1.2p_2)x_2 = m$ . Solving yields  $x_2 = \frac{m}{1.8p_2} = 5$  and  $x_1 = \frac{m}{3p_1} = 3$ . The amount of tax collected by the government is  $0.2p_2x_2 = 10$ . Alice's utility, in turn, is  $x_1x_2^2 = 75$ .

3.3 The second scheme is recommended to the government because the government can collect the same revenue with less distortion to the agent's consumption, and hence with a higher utility for the agent. In particular, with this Cobb-Douglas utility function,  $x_2$  is more important relative to  $x_1$ . The tax rate required on good 2 is less than that for good 1 in order to generate the same revenue, and taxing  $x_2$  causes a smaller reduction in the consumer's utility.