Microeconomics

Question 1. Consider the utility function

$$u(x_1, x_2) = 10x_1 - x_1^2 + x_2,$$

again with accompanying budget constraint $p_1x_1 + p_2x_2 = m$.

1.1 *[5 points]* Define monotonic preferences. For what bundles (x_1, x_2) does this utility function represent monotonic preferences?

1.2 *[10 points]* Form the Lagrange function associated with this utility maximization problem, find the first-order conditions, and solve for the demand function for good x_1. Do not worry about second-order conditions.

1.3 *[15 points]* Find the own price elasticity of demand for good 1. Now assume $p_2 = 1$. When (i.e., for what prices p_1) is demand inelastic? unit elastic? elastic? With this answer in mind, draw a graph with p_1 on the horizontal axis. Identify those values of p_1 for which demand is inelastic, unit elastic, and elastic. Now put the amount of money the consumer spends on good x_1 (i.e., p_1x_1) on the vertical axis. Using what you know about the elasticity, show where the resulting function is increasing, where it is decreasing, where it achieves its maximum, and where it hits the horizontal axis.

Question 2. Consider the utility function

$$u(x_1, x_2) = (x_1 - 2)(x_2 - 3),$$

with accompanying budget constraint $p_1x_1 + p_2x_2 = m$.

2.1 *[5 points]* Draw an indifference curve for this utility function.

2.2 *[10 points]* Form the associated Lagrange function and find the associated first-order conditions for maximizing this utility function subject to the constraint.

2.3 *[10 points]* Find the Bordered Hessian for this constrained maximization problem. State the sufficient second-order conditions and verify that they hold.

2.4 *[15 points]* Solve the first-order conditions to find the demand function for good x_1. Are goods x_1 and x_2 substitutes or complements? (To answer this question, take the appropriate derivative and see whether it is positive or negative.) Does consumption of x_1 increase or decrease in income. (Again, answer with a derivative.)

Question 3. Suppose preferences are characterized by the utility function $u(x_1, x_2) = x_1 x_2^2$. As usual, the budget constraint is $p_1 x_1 + p_2 x_2 = m$. To be concrete, suppose $p_1 = p_2 = 10$ and $m = \$90$.

3.1 *[10 points]* Now suppose the government places a 50% tax on x_1. Write the new budget constraint, given this tax. Identify the utility maximizing bundle of x_1 and x_2 for Alice, the amount of tax revenue collected by the government, and calculate Alice's utility. Ignore second-order conditions.

3.2 *[10 points]* Now suppose instead the government places a 20% tax on x_2. Once again, write the new budget constraint, given this tax. Identify the utility maximizing bundle of x_1 and x_2 for Alice, the amount of tax revenue collected by the government, and calculate Alice's utility. Ignore second-order conditions.

3.3 *[10 points]* In light of your answers to [3.1] and [3.2], which of the two tax policies considered in those parts would you recommend to the government? Why?