

# Microeconomics

## Question 1.1

The profit functions are:

$$\pi_1 = (14 - 2x_1 - 2x_2)x_1 - 2x_1,$$

$$\pi_2 = (14 - 2x_1 - 2x_2)x_2 - 2x_2.$$

## Question 1.2

The new profit functions are:

$$\pi_1 = (12 - 2x_1 - 2x_2)x_1,$$

$$\pi_2 = (12 - 2x_1 - 2x_2)x_2.$$

These profit functions are identical to those in [1.1].

## Question 1.3

Questions [1.1] and [1.2] indicate that if marginal cost is constant, then we can simplify our analysis without altering the problem by assuming there are no costs and defining a new demand function that we can interpret as identifying “price net of marginal costs.”

## Question 1.4

The first order conditions for profit maximization are:

$$12 - 4x_1 - 2x_2 = 0,$$

$$12 - 2x_1 - 4x_2 = 0.$$

The solution to this system of equations is  $x_1 = 2$  and  $x_2 = 2$ . The firms will earn profits  $\pi_1 = 8$  and  $\pi_2 = 8$ .

### Question 1.5

A monopoly that faces demand function  $p = 12 - 2x$  and no costs has profit function  $\pi = (12 - 2x)x$ . This profit function is maximized at  $x = 3$ , giving a profit of 18.

### Question 1.6

If each firm produces half the monopoly output, then  $x_1 = 3/2$  and  $x_2 = 3/2$ . This gives price  $p = 6$  and overall profit of  $\pi = 18$ . The firms will earn profits  $\pi_1 = 9$  and  $\pi_2 = 9$ .

### Question 1.7

If firm 2 is producing half the monopoly output ( $x_2 = 3/2$ ), then firm one wants to produce the amount that maximizes

$$(12 - 2x_1 - 2(3/2))x_1.$$

This equation is maximized when  $x_1 = 9/4$  which is more than half the monopoly output. Given that firm 2 has reduced its output from the duopoly level to half the monopoly level, firm 1 would like to expand output.

### Question 1.8

If firm 1 cooperates and produces half the monopoly output in each period, we know from [1.5] that 1 will earn 9 in each period. Firm 1 will thus have total profit of  $9N$ . If firm 1 deviates in the first period it will produce  $9/4$  as in [1.6]. This gives firm 1 a profit of  $(12 - 2(9/4) - 2(3/2))(9/4)$  for period 1. After period 1, both firms will produce as in [1.4] and earn a profit of 8. So firm 1 will receive a total profit of  $(12 - 2(9/4) - 2(3/2))(9/4) + 8(N - 1)$ . Firm 1 should produce half the monopoly output in the first year if

$$9N > (12 - 2(9/4) - 2(3/2))(9/4) + 8(N - 1).$$

For sufficiently large  $n$ , this inequality will hold.

### Question 1.9

In the situation described in [1.8], the larger the number of interactions  $N$ , the more likely the two firms will cooperate in every period. However, if the firms are allowed to follow more sophisticated strategies, then a finite number of

repeated interactions is unlikely to produce collusion. Imagine the firms are in the last year of their interaction. What benefit will firm 1 get from cooperating when they can cheat on the agreement, make extra profit and face no retaliation because there is no more interaction? The setup of question [1.8] doesn't allow firms to follow this strategy, but realistically they could. After realizing that it is of no benefit to cooperate in the last period of interaction, it is obvious that firms should not cooperate in any period of interaction by backward induction. However, if the firms interact with each other an infinite number of times, it is possible that cooperation can be mutually beneficial.

### Question 2.1

The firm has constant costs.

### Question 2.2

A competitive equilibrium is a set of prices and an allocation such that all consumers are maximizing their utility subject to their budget constraints, all firms are maximizing their profit subject to their production technology and market prices, and all markets clear (or equivalently, aggregate excess demand for each good is zero). In this particular setting, the agent chooses  $l$  and  $c$  in order to maximize the utility function. The firm chooses the amount of labor to employ in order to maximize profits. Finally, the consumer must optimally consume all the firm's production and the firm must hire all the consumer's labor, in order for markets to clear.

### Question 2.3

The firm's profit-maximization problem is:

$$\text{Max}_{T-l} \quad k(T-l) - w(T-l)$$

If  $k > w$ , then profits will be infinitely large and the labor market will not clear. If  $k < w$ , then production will be zero, making impossible for the output market to clear (since the consumer's utility is  $-\infty$  when the consumer consumes nothing). Therefore,  $k = w$ . In this case, the equilibrium profits of the firm are zero. We expect this because the firm has constant costs.

### Question 2.4

The consumer's budget constraint is  $c = w(T-l)$ . The consumer's utility maximization problem is:

$$\text{Max} \ln(c) + \ln(l),$$

$$s.t. \quad c = w(T - l).$$

This leads to the Lagrangian:

$$\mathcal{L} = \ln(c) + \ln(l) + \lambda(c - w(T - l)).$$

The first order conditions are:

$$\begin{aligned} \frac{1}{c} + \lambda &= 0, \\ \frac{1}{l} + w\lambda &= 0, \\ c - w(T - l) &= 0. \end{aligned}$$

Solving this system of equations leads to  $l = \frac{T}{2}$  and  $c = \frac{wT}{2}$ .

### Question 2.5

Pareto efficiency means that there is no alternative allocation such that all consumers have at least the same level of utility and at least one consumer has greater utility. The competitive equilibrium of this economy is  $p = 1$ ,  $w = k$ ,  $c = \frac{kT}{2}$  and  $l = \frac{T}{2}$ . This competitive equilibrium is Pareto efficient. On the one hand, this follows directly from the first welfare theorem. Alternatively, it is straightforward to calculate that the consumer's marginal rate of substitution is equal to the firm's marginal rate of technical substitution, which suffices for efficiency.

### Question 2.6

For there to be unemployment or underemployment, labor must be in excess supply. This is not the case here, since a competitive equilibrium requires zero excess demand and supply.

### Question 3.1

If the value of the company is 0, then the current owners will accept a bid for any amount. If the value of the company is 25, the current owners will accept any bid of 25 or above. If the value of the company is 100, the current owners will accept any bid of 100 or above.

### Question 3.2

For bids less than 25, the firm will only accept if and only if it is worth 0. For bids less than 100 and greater than or equal to 25, the firm will accept if and only if it is worth 0 or 25. For bids greater than or equal to 100 the firm will always accept. The expected value to the buyer of a firm accepting a bid in the range  $0 \leq B < 25$  is 0. The expected value to the buyer of a firm that accepts a bid in the range  $25 \leq B < 100$  is  $50/2$ . The expected value to the buyer of a firm accepting a bid in the range  $100 \leq B$  is  $250/3$ .

### Question 3.3

You should place a bid of 0 or 25. Both of these bids have an expected profit of zero while all other bids have negative expected profit. Notice, however, that bidding 25 carries the risk that the company is worthless and therefore you will suffer a loss, while bidding 0 does not.

### Question 3.4

If you know the value of the firm, then you will bid that value of the firm (or arbitrarily close to it) and your bid will be accepted. The value of this information is thus the expected profit you will receive from knowing the firm's value before your bid. If the value of the firm is 0, then you get no benefit from knowing this before you bid. If the value of the firm is 25 and you learn this before you bid, you will receive profit of 25; and if the value is 100 you will receive profit 100. Therefore, the expected profit from knowing the value of the company before you bid is  $\frac{1}{3}(0 + 25 + 100)$ , and this is the most you would pay to learn the value of the firm before you bid.

### Question 3.5

It is likely to be attractive for an outsider to buy a firm if the value of the firm to the outsider is much greater than the value of the firm to the current owners, or if the outsider has a very precise idea of the value of the firm.