

Microeconomics

Question 1

- 1.1 An individual with a utility function $u(\cdot)$ is risk averse if $u(\pi_1 x_1 + \pi_2 x_2) > \pi_1 u(x_1) + \pi_2 u(x_2)$. In this case, this is

$$\sqrt{\pi_1 x_1 + \pi_2 x_2} > \pi_1 \sqrt{x_1} + \pi_2 \sqrt{x_2}.$$

Squaring both sides, this is

$$\pi_1 x_2 + \pi_2 x_2 > \pi_1^2 x_1 + 2\pi_1 \pi_2 \sqrt{x_1 x_2} + \pi_2^2 x_2.$$

Rearranging, this is

$$\pi_1(1 - \pi_1)x_1 - 2\pi_1 \pi_2 \sqrt{x_1 x_2} + \pi_2(1 - \pi_2)x_2 > 0.$$

But, now noticing that $\pi_1 = 1 - \pi_2$, we can eliminate the probabilities altogether, at which point this is

$$(\sqrt{x_1} - \sqrt{x_2})^2 > 0,$$

which is obvious. Nonetheless, this is a rather lengthy argument for an exam. Other acceptable answers are to note that this person is risk averse if $u(\cdot)$ is strictly concave, or equivalently, its second derivative $u''(\cdot)$ is negative. This person is risk averse since $u(x) = \sqrt{x}$ has a negative second derivative.

- 1.2 The expected utility from investing in the fund is $0.8\sqrt{100(1+0.9r)} = 8\sqrt{1+0.9r}$. The expected utility for not investing is $\sqrt{100} = 10$. In order for him to be willing to invest in the fund, we require $8\sqrt{1+0.9r} > 10$. This is as far as one needed to go on the exam. Continuing, this is $1+0.9r > (\frac{5}{4})^2$, i.e., $r > \frac{5}{8}$.
- 1.3 The mean return per dollar of the two funds are: $0.8 \cdot 0.6 = 0.48$ and $0.7 \cdot 0.7 = 0.49$, respectively. The fund manager would like to choose the second one.

The investor's expected utility from investing in plan 1 is $0.8\sqrt{100(1+0.9 \cdot 0.6)}$, while his expected utility from investing in plan 2 is $0.7\sqrt{100(1+0.9 \cdot 0.7)}$. The investor would find the first plan better than the second. The intuition behind this is that the two funds give expected values that are very close, while the first one is less risky (since the probability of a zero outcome is only .2). You were not required to, but the following calculation verifies this intuition:

$$\begin{aligned}
 0.8\sqrt{100(1+0.9\cdot 0.6)} &> 0.7\sqrt{100(1+0.9\cdot 0.7)} \\
 \Leftrightarrow 8\sqrt{1+0.9\cdot 0.6} &> 7\sqrt{1+0.9\cdot 0.7} \\
 \Leftrightarrow \left(\frac{8}{7}\right)^2 &> \frac{1+0.9\cdot 0.7}{1+0.9\cdot 0.6} = \frac{1.63}{1.54} \\
 \Leftrightarrow \frac{64}{49} > \frac{163}{154} &\Leftrightarrow \frac{64}{49} - 1 > \frac{163}{154} - 1 \\
 \Leftrightarrow \frac{15}{49} > \frac{9}{154}, &
 \end{aligned}$$

which is obvious. This gives us a comparison between these two funds. One might also want to check whether either fund is better than not investing, but this was not required.

Question 2

Question 2.1 The cost minimization problem is:

$$\min_{x_1, x_2} p_1x_1 + p_2x_2 \quad \text{subject to} \quad q = x_1x_2$$

The first order conditions are:

$$\begin{aligned}
 p_1 + \lambda x_2 &= 0 \\
 p_2 + \lambda x_1 &= 0 \\
 q &= x_1x_2
 \end{aligned}$$

The conditional demand functions are:

$$\begin{aligned}
 x_1^c &= \sqrt{\frac{p_2q}{p_1}} \\
 x_2^c &= \sqrt{\frac{p_1q}{p_2}}
 \end{aligned}$$

Question 2.2

The cost function is:

$$\begin{aligned}
 C(q) &= p_1x_1^c + p_2x_2^c \\
 &= p_1\sqrt{\frac{p_2q}{p_1}} + p_2\sqrt{\frac{p_1q}{p_2}} \\
 &= \sqrt{p_1p_2q} + \sqrt{p_1p_2q} \\
 &= 2\sqrt{p_1p_2q}
 \end{aligned}$$

Question 2.3

This firm exhibits decreasing costs. You can see this in two ways. The first is through the cost function:

$$C(tq) = 2\sqrt{p_1 p_2 (tq)} = t^{\frac{1}{2}} 2\sqrt{p_1 p_2 q} = t^{\frac{1}{2}} C(q) < tC(q)$$

The other way is to notice that the production function has increasing returns to scale:

$$f(tx_1, tx_2) = (tx_1)(tx_2) = t^2 x_1 x_2 = t^2 f(x_1, x_2) > t f(x_1, x_2)$$

Increasing returns to scale means the firm has decreasing costs.

Question 2.4

To show that marginal cost is given by the Lagrange multiplier, λ , take the derivative of the cost function with respect to q :

$$\frac{dC}{dq} = \sqrt{\frac{p_1 p_2}{q}}$$

To compare this to the Lagrange multiplier, solve the first order condition for λ and plug in the conditional demand function for x_2^c :

$$\lambda = -\frac{p_1}{x_2^c} = -\frac{p_1}{\sqrt{\frac{p_1 q}{p_2}}} = -\sqrt{\frac{p_1 p_2}{q}}$$

In this case, the marginal cost is equal to the negative Lagrange multiplier because of how we set up the constraint. You can also solve this problem in the general case if you differentiate $C(q) = p_1 x_1^c + p_2 x_2^c$ with respect to q :

$$\begin{aligned} \frac{dC}{dq} &= p_1 \frac{dx_1^c}{dq} + p_2 \frac{dx_2^c}{dq} \\ &= \lambda \left[\frac{df}{dx_1} \frac{dx_1^c}{dq} + \frac{df}{dx_2} \frac{dx_2^c}{dq} \right] \\ &= \lambda \end{aligned}$$

where we get the second inequality by substituting the first-order conditions from the cost minimization problem. We can see that the term in brackets equals 1 if we differentiate the expression $f(x_1, x_2) = q$ with respect to q .

Question 2.5

Since the firm exhibits decreasing costs and faces a fixed price (in a competitive market), any firm that enters the market is going to produce either zero or an arbitrarily large amount of output. Suppose it is the latter. Then the price in the market will decrease, forcing firms to leave the market, until we have so few firms that the market is no longer perfectly competitive.

Question 3

1. Wealth in each state is:

State 1: $w_1 - zP$.

State 2: $w_2 - zP + zC$.

Expected utility is:

$$EU = \pi_1 u(w_1 - zP) + \pi_2 u(w_2 - zP + zC).$$

2. The first-order condition is:

$$\frac{dEU}{dz} = \pi_1 u'(w_1 - zP)(-P) + \pi_2 u'(w_2 - zP + zC)(-P + C) = 0.$$

3. Manipulating the first-order condition gives:

$$\frac{u'(w_1 - zP)}{u'(w_2 - zP + zC)} = \frac{\pi_2}{\pi_1} \frac{(-P + C)}{P}.$$

If consumption is the same in both states, then $\frac{u'(w_1 - zP)}{u'(w_2 - zP + zC)} = 1$. Therefore $\frac{\pi_2}{\pi_1} \frac{(-P + C)}{P} = 1$. This is equivalent to $P = \pi_2 C$.

If consumption in state 1 is greater than in state 2, then $u'(w_1 - zP) < u'(w_2 - zP + zC)$ (assuming u is concave). This gives $\frac{u'(w_1 - zP)}{u'(w_2 - zP + zC)} < 1$, $\frac{\pi_2}{\pi_1} \frac{(-P + C)}{P} < 1$ and $\pi_2 C < P$. A similar argument shows that when consumption in state 2 is greater than wealth in state 1, $\pi_2 C > P$. Notice in particular that a key point was to link this answer to the conditions determining consumer optimality.