Microeconomics

Question 1. Consider an expected utility maximizer, with a utility function u over certain monetary payoffs given by $u(x) = \sqrt{x}$, with an income of 100.

1.1 [*10 points*] Define what it means to be risk averse. Is a person with this utility function risk averse? Why? Be precise.

1.2 [*10 points*] Suppose this person has the option of putting all of her money into an investment fund (to simplify the problem, partial investments are not allowed). With probability .8, the fund is a success, and her payoff is $100(1 + .9r)$. To interpret this payoff, think of r as the return the fund earns when it is a success, with the fund keeping $.1r$ (i.e., ten percent) as a commission for managing her money and paying $.9r$ (as well as her original 100) to her. With probability .2, the fund fails and she loses all her money (and the fund earns no commission). How large must r be in order for this person to be willing to invest in the fund? (Once you have an expression identifying this value, you need not simplify it.)

1.3 [*10 points*] Now suppose the fund manager has a choice of two investments for this person's money. One is a success with probability .8, giving a return or $r = .6$ (and with probability .2 is a failure, causing the person to lose all her money). The other is a success with probability .7, giving a return $r = .7$ (and with probability .3 is a failure, causing her to lose all her money). Given that it wants to maximize its expected commission, which investment will the fund choose? Which expression will this person choose? To answer this last question, write an expression that must hold if the person is to choose the first investment, and an analogous expression that must hold if she is to choose the second. You need not simplify either expression. Then, without doing algebra or simplifying, explain which of these inequalities you expect to hold, and hence which investment the person will prefer, and why.

Question 2. Consider a firm that hires quantities x_1 and x_2 of inputs 1 and 2, at prices p_1 and p_2. The production function is given by $f(x_1, x_2) = x_1 x_2$.

2.1 [*15 points*] Form the cost-minimization problem for this firm (assume all inputs are variable), find the associated first-order conditions, and the conditional demand functions.

2.2 [*5 points*] Find the cost function for this firm.

2.3 [*5 points*] Indicate whether this firm exhibits increasing, decreasing, or constant costs. Be precise in justifying your answer.

2.4 [*10 points*] Show that marginal cost is given by the Lagrange multiplier (or its negative, depending on how you set up your constraint). You may answer this question either by doing the calculations for this particular case, or by presenting a precise general argument.

2.5 [*5 points*] Now suppose the market in which this firm produces is perfectly competitive. What long-run adjustments would you expect to see. Why?

Question 3. Consider an individual whose utility function over monetary lotteries is given by $U(x_1, x_2, \pi_1, \pi_2) = \pi_1 u(x_1) + \pi_2 u(x_2)$.

3.1 [*10 points*] Suppose this this person has incomes (w_1, w_2) in states 1 and 2, with $w_1 > w_2$. Suppose she is offered an insurance policy of the form (zP, zC), where she has to choose the level of insurance z. In particular, this policy requires a premium zP to be paid in both states, and plays a claim zC in state 2. Choosing $z = 0$ corresponds to buying no insurance at all, while larger values of z entail larger premia and larger payments in the event of a loss. For a fixed z, identify this person's consumption in state 1 and consumption in state 2. Given this, write this person's expected utility as a function of z.

3.2 [*5 points*] Using your expression for expected utility from [3.1], find the first-order condition determining z.

3.3 [*15 points*] Using your first-order condition from [3.2], when (i.e., for what values of P and C) will this person choose a value of z that gives higher consumption in state 1 than in state 2? When will the reverse occur? When will she choose a value of z that gives her equal consumption in states 1 and 2?