Microeconomics

Question 1. Consider the utility function

$$u(x_1, x_2) = x_1 - \frac{1}{2}x_1^2 + x_2,$$

with budget constraint $p_1x_1 + p_2x_2 = I$.

1.1 *[5 points]* Form the Lagrange function associated with this utility maximization problem and find the first-order conditions. (Do not worry about the fact that this utility function is decreasing in x_1 for large values of x_1; simply proceed as if prices and income are such that the solution is interior.)

1.2 *[10 points]* Find the Bordered Hessian for this constrained maximization problem. State the sufficient second-order conditions. Once you have stated these second-order conditions in the form of inequalities involving determinants, you need proceed no further. You do not have to evaluate the determinants.

1.3 *[10 points]* Find the own price elasticity of demand for good 1. For what values of p_1 is the demand for good 1 elastic, and for what values is it inelastic?

Question 2. Consider the utility function $U(x_1, x_2) = (x_1 - 10)x_2$. Let us interpret x_1 as "health" (or money spent on health) and x_2 as money spent on other goods. Let the budget constraint be $x_1 + x_2 = I$.

2.1 *[10 points]* The government is considering a health care plan. In one proposal, the government gives each individual an amount S (for subsidy). The intention is presumably that people use this money to purchase health care, but no such restrictions are imposed—people are free to spend S however they want. What is the effect on purchases of good 2?

2.2 *[15 points]* Suppose instead the government subsidizes the purchase price of health care, by paying proportion t of what the individual pays for health care. Hence, the individual's price is $(1 - t)$ per unit of health care and the cost to the government of this plan is tx_1, where

x_1 is the amount of health care the person consumes. Suppose the subsidy t is chose so that this program costs the government the same as the direct transfer program described in [2.1]. Assuming (as usual) that the individual maximizes utility in both cases, which program will result in a higher utility for the individual? Be as precise as you can in justifying your answer.

Question 3. Consider a person who has a total of T units of time that the person can allocate to leisure (h), *time spent consuming* (kc), and working (whatever time ($T - h - kc$) is left after leisure and time spent consuming). This person is paid w per unit of time worked, and hence has an income of $w(T - h - kc)$, which the person spends on consumption (c). The price of consumption is 1 and the person has no other income. The new element in this problem is the time spent consuming: if the consumption is c, then the person must spend kc units of time doing the consuming, where k is a parameter. Utility is given by $U(c, h) = ch$.

3.1 *[10 points]* Formulate the budget constraint for this person. Start with a constraint of the form $c = \ldots$, with expenditure c on one side and income on the other.

3.2 *[5 points]* Find the demand function for consumption.

3.3 *[5 points]* Now suppose k decreases. What happens to the optimal consumption of c. Does the common saying "time is money" make sense?

Question 4. Consider a person who chooses consumption x_1 and x_2 in the current (x_1) and future (x_2) period to maximize the utility function $U(x_1, x_2) = x_1^2 x_2$ subject to the budget constraint $x_1 + \frac{x_2}{1+r} = I_1$. (Notice that this person has no income in the second period.)

4.1 *[5 points]* Find the demand function for good 1.

4.2 *[10 points]* Find the compensated demand function for good 1.

4.3 *[15 points]* Verify that the Slutsky equation holds for this good. To do this, write the general form of the Slutzky equation, then use the demand and compensated demand functions you have found to substitute in specific terms. You then need simplify no further.