

Microeconomics

1. True/False Questions (TOTAL: 20 points):

In this section, write whether each statement is True or False. Please fully explain your answer, using a diagram if appropriate. No credit will be given for an answer without an explanation.

- (a) (5 points) As long as the marginal cost of production is greater than the average variable cost, then the average variable cost is increasing.

True. By the definition of the average variable cost, we can find that as long as $MC > AC$, $\frac{dAC}{dq} > 0$.

- (b) (5 points) In a perfectly competitive market with constant long run marginal cost, the consumer will bear all the taxation burden.

True. This is because the supply curve is a flat line, no matter what the demand is, supply is inelastic.

- (c) (5 points) In a perfectly competitive market, firms take the market price as a given, which implies that the market demand is infinitely elastic.

False. In a competitive market, firms are assumed to be very small, therefore they are assumed to have no influence on the market price – like if they faced an infinitely elastic demand at the market price level. Nevertheless, the market demand can be elastic or rigid.

- (d) (5 points) In an exchange economy, no individual will ever prefer a point inside the utility possibilities frontier to a point on the utility possibilities frontier.

False. The points on the Pareto frontier are such that one cannot improve the situation of one individual without making someone else worse off – we can move to an interior point and make one individual better off, just not the whole society.

Long Questions:

2. (15 points) Ricardo produces widgets, using as inputs labor (L) and machines (K). His production function is given by the following equation:

$$q = 10K^{2/3} + L^{1/2}$$

- (a) (4 points) What type of returns to scale (increasing/constant/decreasing) does Ricardo's production function exhibit?

The production function exhibits decreasing returns to scale:

$$F(2K, 2L) = 10(2K)^{2/3} + (2L)^{1/2} = 10 \cdot 2^{2/3} K^{2/3} + 2^{1/2} L^{1/2} < 2 \cdot [10K^{2/3} + L^{1/2}] = 2F(K, L)$$

At the end of last year, Ricardo bought his only machine for \$1,000. He will use this machine for 5 years, after which the machine will have no value. Ricardo will calculate depreciation linearly (depreciation will be 20% of the total value of the machine per year). This machine has no other use besides Ricardo's production of widgets, and, at this moment, Ricardo cannot buy any more machines.

- (b) (4 points) What is Ricardo's annual fixed cost of production? Is the fixed cost sunk or not? Explain.

$$FC = 0.2 * 1000 = 200$$

All the fixed cost is sunk, as Ricardo already paid it in the past and cannot recover it. There is no opportunity cost, as the machine has no other use besides Ricardo's production of widgets.

- (c) (4 points) What is Ricardo's demand for labor as a function of the quantity he wants to produce annually?

$$q = 10 * K^{2/3} + L^{1/2}$$

$$L^d = \begin{cases} (q - 10)^2, & \text{if } q \geq 10 \\ 0, & \text{if } q < 10 \end{cases}$$

- (d) (3 points) Assuming that wage equals 1, what is Ricardo's annual total cost function?

$$TC = \begin{cases} q^2 - 20q + 300, & \text{if } q \geq 10 \\ 200, & \text{if } q < 10 \end{cases}$$

3. (10 points) Sally's firm produces granola bars with a fixed cost of 10 (this cost is already sunk). Her variable cost function is $VC = q^2 + 2q$.

- (a) (4 points) Assuming the market for granola bars is competitive, derive Sally's supply function?

$$\text{Supply: } P = MC \leftrightarrow P = 2q^s + 2$$

- (b) (6 points) What is Sally's surplus if the market price is 6? What is her profit? Does she want to stay in this market? Explain.

$$\text{If } P = 6, q = 2. \text{ Producer surplus} = \text{Revenue} - \text{Variable Costs} = 12 - 8 = 4.$$

$$\text{Profit} = \text{Revenue} - \text{Total Costs} = 12 - 10 - 8 = -6.$$

Sally would want to stay in the market, even though she makes a loss, as that will minimize her losses – she will be able to cover all her variable costs and still make some extra money, which will lower the loss she would have due to her sunk costs.

4. (24 points) Suppose the demand function for corn is $Q_d = 10 - 2p$, and supply function is $Q_s = 3p - 5$. The government is concerned that the market equilibrium price of corn is too low and would like to implement a price support policy to protect the farmers. By implementing the price support policy, the government sets a support price and purchases the extra supply at the support price. In this case, the government sets the support price $p_s = 4$.

- (a) (4 points) Calculate the original market equilibrium price and quantity in absence of the price support policy.

The original equilibrium price is $p_0 = 3$. And the equilibrium quantity is 4.

- (b) (3 points) At the support price $p_s = 4$, find the quantity supplied by the farmers, the quantity demanded by the market, and the quantity purchased by the government.

When the price is $p_s = 4$, the demand is 2, and supply will be 7, so the government need to buy 5.

- (c) (3 points) Draw a diagram to show the change in the producer surplus due to the implementation of the price support policy. Calculate the change in the producer surplus.

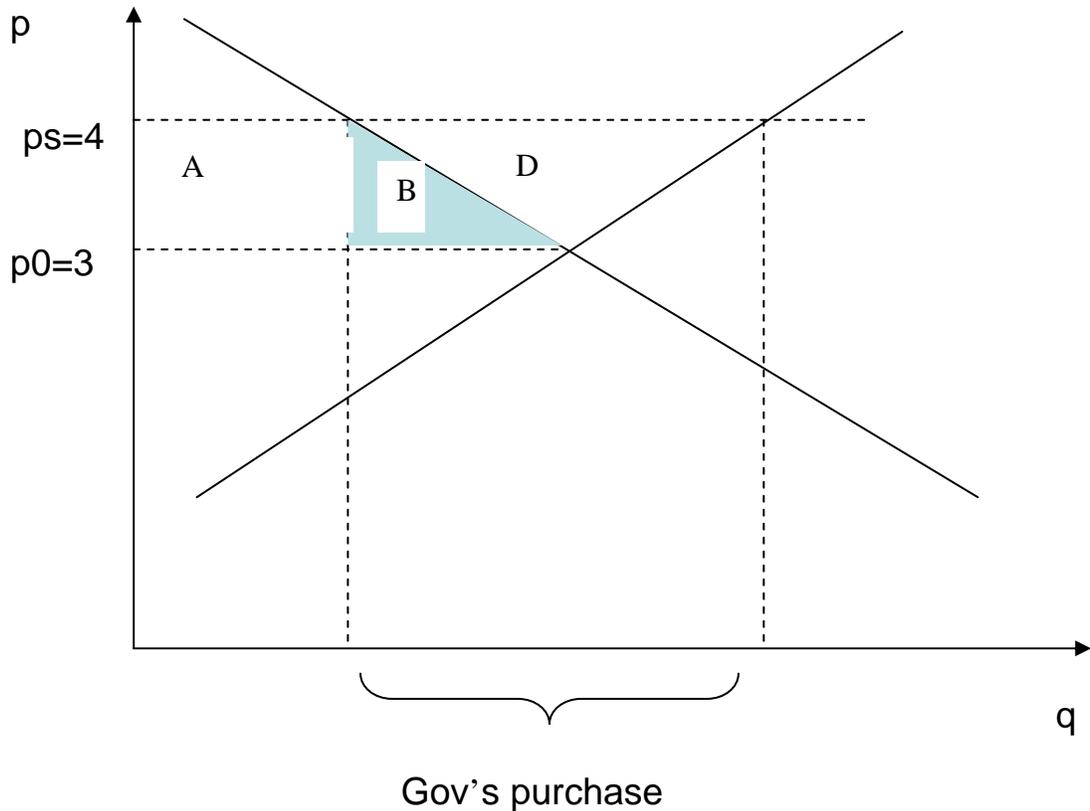
The producer's gain is

$$1 \times 7 - \frac{1}{2}(7 - 4) = \frac{11}{2}$$

Which is area A+B+D in the following graph.

- (d) (3 points) Draw a diagram to show the change in the consumer surplus due to the implementation of the price support policy. Calculate the change in the consumer surplus.

The loss to consumer is $2(4 - 3) + (4 - 2)(4 - 3)/2 = 3$, which is area A+B in the following graph.



- (e) (3 points) Calculate the cost to the government to implement the price support policy. Draw a diagram to show the government cost.

*Cost to the government: $P_s Q_g = 20$, which is the rectangle area indicated in the above graph by Gov's purchase * price.*

- (f) (8 points) Suppose now the government switches from price support policy to subsidy policy. For each unit of corn produced, the government subsidizes the farmer $s = \frac{5}{3}$. Find the new equilibrium price under this subsidy policy. How much money will the government have to spend in order to implement this subsidy policy?

Under subsidy $s = \frac{5}{3}$, the suppliers face the price $p_b + \frac{5}{3}$, and the consumers face the price p_b , thus,

$$10 - 2p_b = 3(p_b + s) - 5$$

This implies

$$p_b = 2$$

The new amount of supply is $Q = 10 - 2p_b = 6$, which is the amount that the government needs to subsidize. The government's total budget is $sQ = 10$.

5. (15 points) Molly's company produces knee warmers according to the following production function:

$$q = (K-8)^{1/4} L^{1/4}$$

- (a) (4 points) Assuming that the unit cost of capital (r) and the unit wage (w) are both equal to 1, derive Molly's demand for inputs—capital and labor, respectively—as a function of her choice of output (q).

Cost minimization will lead to the following two conditions:

$$\begin{cases} \frac{K-8}{L} = \frac{1}{1} \\ q = (K-8)^{1/4} L^{1/4} \end{cases} \Leftrightarrow \begin{cases} L^d = q^2 \\ K^d = 8 + q^2 \end{cases}$$

- (b) (2 points) Show that Molly's long run total cost function is given by $C(q) = 8 + 2q^2$.

$$C(q) = C(K^d, L^d) = 8 + 2q^2$$

The demand for knee warmers is given by $P = 40 - Q^d$. There are no costs of entry or exit for a firm on the market for knee warmers. Any firm in this market will have access to the same technology as Molly.

- (c) (6 points) What will the price be in the long run in this market? How much will each firm produce in this market in the long run.

In the long run, $P = \text{Min LAC}$

Min LAC: $LAC = LMC$

$$\frac{8}{q} + 2q = 4q$$

$$q^{\text{Min LAC}} = 2$$

Price in the long run = Min LAC = 8

- (d) (3 points) How many firms will there be in this market in the long run?

If Price = 8, looking at the demand function, we see that $Q = 32$, which implies that on the long run, we will have 16 firms in this market.

6. (15 points) Consider a two person economy consisting of Ann and Bob only. Both of them only consume x and y . Ann's utility over these two goods is $U_A(x_A, y_A) = x_A y_A^2$ and Bob's utility is $U_B(x_B, y_B) = x_B^2 y_B$. Initially, Ann is endowed with 9 units of x , zero units of y , and Bob is endowed with 6 units of y , zero units of x .

- (a) (2 points) Write Ann's marginal rate of substitution in terms of x_A and y_A and Bob's marginal rate of substitution in terms of x_B and y_B .

$$MRS_A = \frac{MU_x}{MU_y} = \frac{y_A}{2x_A}$$

$$MRS_B = \frac{MU_x}{MU_y} = \frac{2y_B}{x_B}$$

- (b) (5 points) Derive the equation for the contract curve.

Let $MRS_A = MRS_B$, we have

$$\frac{y_A}{2x_A} = \frac{2y_B}{x_B} = \frac{2(6 - y_A)}{9 - x_A}$$

thus

$$y_A = \frac{24x_A}{9 + 3x_A} = \frac{8x_A}{3 + x_A}; \text{ alternatively, one can write}$$

$$\frac{2y_B}{x_B} = \frac{6 - y_B}{2(9 - x_B)} \text{ thus } y_B = \frac{2x_B}{12 - x_B}$$

- (c) (8 points) Find the general equilibrium allocation of x and y among Ann and Bob of the above economy

For Ann, her utility maximization problem is

$$\max x_A y_A^2$$

$$\text{s.t. } x_A + p y_A = 9$$

it's ready to give the solution $x_A^* = 3$;

For Bob, his utility maximization problem is

$$\max x_B^2 y_B$$

$$\text{s.t. } x_B + p y_B = 6p$$

this gives the solution $y_B^* = 2$;

Thus the final solution for this is:

$$x_A^* = 3; y_A^* = 4; x_B^* = 6, y_B^* = 2; p = \frac{3}{2}$$