

Microeconomics

1. True/False Questions (TOTAL: 20 points):

In this section, write whether each statement is True or False. Please fully explain your answer, using a diagram if appropriate. No credit will be given for an answer without an explanation.

(a) (5 points) A risk averse individual that has to decide between two different lotteries will always prefer a lottery with less risk.

False. A risk averse individuals dislike risk but not at “any price”. They will take into account the expected value of each lottery in their decisions.

(b) (5 points) Steven only consumes two goods: X and Y. If X is a Giffen good for Steven, then Y must be a normal good for Steven.

True. A Giffen good is an inferior good. Since Steven only consumes two goods, they cannot both be inferior goods. Therefore, Y is a normal good. (Need to point out explicitly that Giffen goods are a special kind of inferior goods, or the negative income effect of a Giffen good dominates the substitution effect. People who only say that the consumption of a Giffen good decreases with income without explanation lose partial points.)

(c) (5 points) Ann and Bob consume clothes (C) and food (F) only. Ann’s utility function is $U(C, F) = a_A \ln C + b_A \ln F$, and Bob’s utility function is $U(C, F) = a_B \ln C + b_B \ln F$. $a_A, b_A, a_B, b_B > 0$. The price of clothes is P_C , and the price of food is P_F . Ann and Bob must have the same marginal rates of substitution (MRS) of clothes for food at the optimal level of consumption.

True. With utility functions in this form, we always have an interior solution, and hence Ann’s MRS and Bob’s MRS both equal the price ratio.

(d) (5 points) If S and F are perfect complements you must be indifferent between these two bundles:

- i. One unit of S and one unit of F
- ii. Two units of S and one unit of F

False. The line that goes through the kinks of the indifference curves doesn’t need to have slope 1. Consider a consumer that likes to consume both goods in a ratio 2:1. In that case the first bundle is in another indifference curve with a lower level of utility associated.

2. Short Answer Questions (TOTAL: 10 points):

(a) (4 points) Mary's demand curve for food is $Q = 10 - 2P$. Her price elasticity of demand for food at price P^* equals $-\frac{2}{3}$. How much is P^* ?

Mary's demand function is $Q_B = 10 - 2P$. Her price elasticity of demand at P^* is

$$\frac{dQ_B}{dP} \frac{P^*}{Q_B^*} = -2 \times \frac{P^*}{10 - 2P^*} = -\frac{2}{3}.$$

$$\Rightarrow 3P^* = 10 - 2P^*$$

$$\Rightarrow P^* = 2$$

(b) (6 points) Ann and Bob are a couple. They are the only people in the family. Bob's inverse demand curve for shirts is $P = 5 - \frac{1}{2}Q_B$. Ann's inverse demand curve for shirts is $P = 10 - 2Q_A$. What is their family demand function for shirts? Calculate their family consumption of shirts when the price is 4 and 6, respectively.

Ann's demand function is $Q_A = 5 - \frac{1}{2}P$. When $5 \leq P < 10$, only Ann buys shirts, and when $P < 5$, both of them buy shirts. Therefore, the family demand function is

$$Q = \begin{cases} 5 - \frac{1}{2}P, & \text{if } 5 \leq P < 10 \\ 15 - \frac{5}{2}P, & \text{if } 0 \leq P < 5 \end{cases}. \text{ They consume 5 shirts when } P=4 \text{ and consume 2 shirts}$$

when $P=6$.

Long Questions:

3. (15 points) Jane has utility function over her net income $U(I) = \sqrt{I}$

- (a) (2 points) What are Jane's preferences towards risk? Is she risk averse, risk neutral or risk loving? [Explain briefly your answer]

We know that Jane is risk averse by the concavity of her utility function. She prefers an amount for sure than any lottery that has that amount as expected value but involves some risk.

- (b) (6 points) Jane drives to work every day and she spends a lot of money in parking meters. Many days the thought of cheating and not paying for parking crosses her mind. However she knows that there is a $\frac{1}{4}$ probability of being caught in a given day if she cheats, and that the cost of the ticket is \$36. Her daily income is \$100. What is the maximum amount of she will be willing to pay for one day parking? [Hint: by paying that amount she avoids the risk of getting a ticket!].

She will be indifferent between paying x for parking or facing the "cheating lottery" when the following equation holds:

$$U(100-x) = 0.25*U(100-36)+0.75*U(100)$$

$$\sqrt{100-x} = 0.25*\sqrt{64} + 0.75*\sqrt{100}$$

$$\sqrt{100-x} = 0.25*8 + 0.75*10 = 2 + 7.5 = 9.5$$

$$100-x = 90.25$$

$$x = 9.75$$

- (c) (2 points) Paul also faces the same dilemma every single day. But he has a utility function $U(I)=I$. His daily income is also \$100. What are Paul's preferences towards risk? Is he risk averse, risk neutral or risk loving?

Paul is risk neutral since he has linear utility function over income.

- (d) (5 points) If the price of one day parking is 9.25, will Paul cheat or pay the parking meter? Will Jane cheat or pay the parking meter under this price?

Paul will cheat since:

$$U(100-9.25) < 0.25*U(100-36) + 0.75*U(100)$$

$$90.75 < 16 + 75 = 91$$

Jane will not cheat since the price of one day parking is lower than what she is willing to pay to avoid the risk of getting caught.

4. (25 points) In Country Faraway, cigarettes are forbidden, so people trade cigarettes in a black market. The cigarette demand is $Q_D = 12 - P$, and the cigarette supply is $Q_S = 2P$.

(a) (3 points) Find the equilibrium price and quantity in the black market.

$$P=4, Q=8.$$

(b) (6 points) The government becomes aware of the black market and reinforces the police so that half of the cigarette supply would be seized and destroyed. Under this circumstance, what are the demand and supply functions? What is the new equilibrium price and quantity? Show the change by using a supply and demand diagram.

$$Q_D = 12 - P, Q_S = P, P=6, Q=6.$$

(c) (4 points) How does the consumer surplus change between (a) and (b)?

In (a), the consumer surplus is 32, and in (b), the consumer surplus is 18. The consumer surplus decreases by 14.

(d) (8 points) Suppose that the government changes the policy and legalizes cigarette dealings. Now cigarettes are traded in an open market. However, for every unit of the cigarettes bought, the buyer has to pay tax T to the government. T is equal to the pre-tax price P . What are the demand and supply functions under this circumstance? What are the equilibrium (pre-tax) price and quantity? What is the after-tax price paid by buyers?

$$Q_D = 12 - (P + T) = 12 - 2P, Q_S = 2P, P=3, Q=6. \text{ The after-tax price is 6.}$$

(e) (4 points) Compare (b) and (d). Which policy do consumers prefer? Which policy does the government prefer? Why?

The quantities and the (after-tax) prices paid by buyers are the same in both cases. Therefore, consumers feel indifferent. For the government, the policy in (b) requires extra expenditure on police and the policy in (d) brings tax revenue. Therefore, the government prefers (d).

5. (29 points) Eric receives utility from days spent traveling on vacation domestically (D) and days spent traveling in a foreign country (F) as given by the utility $U(D, F) = DF$. The price of a day spent traveling domestically is \$160 and in a foreign country \$200. Eric's annual budget for traveling is \$8,000.

- (a) (5 points) Find Eric's utility maximizing choice of days traveling domestically and in a foreign country. Find also his utility level from consuming that bundle.

$$MRS = (\partial U / \partial D) / (\partial U / \partial F) = F/D$$

$$P_D / P_F = 160/200$$

$$\Rightarrow F = 4D/5$$

$$160D + 200F = 8,000$$

$$\Rightarrow D^* = 25 \quad F^* = 20$$

$$\Rightarrow U(25, 20) = 500$$

- (b) (6 points) Suppose that the price of domestic traveling increases to \$250 per day. Calling his budget for traveling x , (suppose by now that it is unknown) find the demand for D and F under the new prices as a function of x .

$$MRS = (\partial U / \partial D) / (\partial U / \partial F) = F/D$$

$$P_D / P_F = 250/200$$

$$\Rightarrow F = 5D/4$$

$$250D + 200F = x$$

$$\Rightarrow D^* = x / 500$$

$$\Rightarrow F^* = x / 400$$

- (c) (4 points) Find the income necessary to make Eric reach the same utility level as before the price change.

$$U(D^*, F^*) = [(x / 500) * (x / 400)] = 500$$

$$\Rightarrow x = \sqrt{100,000,000} = 10,000$$

- (d) (2 points) Compute the quantities demanded with the new prices and the income you found in section c.

$$D^c = 10,000 / 500 = 20$$

$$F^c = 10,000 / 400 = 25$$

- (e) (2 points) Compute the quantities demanded with the new prices and the original income.

$$D^{**} = 8,000 / 500 = 16$$

$$F^{**} = 8,000 / 400 = 20$$

- (f) (5 points) Using your previous answers tell us what is the total change in quantity of D due to the price increase in P_D that the consumer experiences and what part of that change is due to income or substitution effects. Give definitions of what income and substitution effects mean.

Initial bundle: $D^* = 25$ $F^* = 20$

Final bundle: $D^{**} = 16$ $F^{**} = 20$

Compensated bundle: $D^c = 20$ $F^c = 25$

\Rightarrow Substitution effect $20 - 25 = -5$

\Rightarrow Income effect $16 - 20 = -4$

We can decompose the change in quantity generated by a price change in two effects. The substitution effect is the variation in quantity generated by a change in the price ratio but holding utility constant. The income effect is the remaining change in quantity and is the result of the change in the purchasing power of the consumer.

- (g) (5 points) Draw a graph showing the income and substitution effects you found.

