Macroeconomics

Partial Answer Key

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements*, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer.

1. If consumers receive utility from government goods, then government spending has no wealth effects.
2. A higher saving/income ratio causes faster long-run economic growth.
3. Risk averse investors will always prefer assets that have a known rate of return.
4. Real business cycle theory accurately describes the current macroeconomic situation: the oil price is up, auto sales are down, exports are booming, house prices are falling, and a credit crunch has whacked the banks.
5. Firms that have constant returns to scale technologies and take factor prices as given cannot make positive profits.
6. The high expected rate of return to investors explains the low personal saving rate during the 1990's stock boom and the 2000's housing boom.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. **Proportional income taxes in simple dynamic model with a government.**

Consider the following economy:

- **Time:** Discrete; infinite horizon
- **Demography:** Continuum of mass 1 of (representative) consumer/worker households, and a large number of profit-maximizing firms, owned jointly by the households.
- **Preferences:** The instantaneous household utility function is $u(c)$ where c is household consumption and $u(.)$ is strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.
- **Technology:** There is a constant returns to scale technology for which labor is the only input so that a firm that hires h units of labor produces zh units of output.
- **Endowments:** Each household has 1 unit of time per period to allocate between work and leisure.
- **Institutions:** There is a government that has to meet an exogenous stream of expenditures, $\{g_t\}$. Government spending is thrown into the ocean. The government can levy taxes and issue bonds in order to meet its expenditure requirement. Taxes are restricted to being proportional to labor income so that in period t, the tax revenue from a household which provides labor services h_t is $\tau_t w_t h_t$, where τ_t is the period t tax rate and w_t is the wage rate. Every period there are markets for labor, government bonds and consumption goods.

(a) Households solve

$$\max_{\{c_t, h_t, s_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$\text{s.t. } c_t = (1-\tau_t) w_t h_t + (1+r_t)\, s_{t-1} - s_t,$$

where s_t is the saving (in bonds) by a household in period t and r_t is the interest rate on bonds.

Clearly,

$$h_t = 1, \quad \text{for all } t$$

(Alternatively, one can simply note that households do not value leisure, so that $h_t = 1$ for all t, and drop h_t from the household's problem.) The intertemporal optimality condition for saving is

$$u'(c_t) = \beta\,(1 + r_{t+1})\, u'(c_{t+1}).$$

Firms solve

$$\max_{h_t^f} \left\{ z h_t^f - w_t h_t^f \right\},$$

and it follows that for any firm that hires a strictly positive (but finite) amount of labor,

$$w_t = z, \quad \text{for all } t.$$

(b) The government's flow (period-by-period) budget constraint is

$$g_t + (1 + r_t)b_{t-1} = b_t + \tau_t w_t h_t.$$

(c) A *competitive equilibrium* is a sequence of prices, $\{w_t, r_t\}$ a sequence of tax rates $\{\tau_t\}$ and an allocation $\{c_t, h_{t,} h_t^f, b_t, s_t\}$ such that: given prices and tax rates, the allocation solves the households' problem and the firms' problem; the government budget constraint holds; and the markets for labor, consumption goods and bonds all clear.

The market clearing conditions are:

$$\begin{aligned} h_t^f &= h_t = 1, \\ b_t &= s_t, \\ z &= g_t + c_t. \end{aligned}$$

Using these the consumer's budget constraint becomes

$$c_t = (1 - \tau_t)z + (1 + r_t)\, b_{t-1} - b_t,$$

the government budget constraint becomes

$$g_t + (1 + r_t)b_{t-1} = b_t + \tau_t z,$$

and we can rewrite the intertemporal optimality condition as

$$1 + r_{t+1} = \frac{u'(z - g_t)}{\beta u'(z - g_{t+1})}.$$

(d) In this model, Ricardian equivalence holds. The timing of τ_t does not affect the equilibrium allocation, which is summarized by $c_t = z - g_t$ for all t.

(e) If the utility function was replaced by $u(c_t, 1-h_t)$, where $u(.,.)$ is strictly increasing in both arguments, the household would care about leisure and the proportional tax would distort the labor-leisure time allocation. Goods market clearing will be $zh_t = g_t + c_t$ and h_t will depend on the timing of taxes. The equilibrium wage will still be z for all t but the first-order condition for the labor leisure choice will imply

$$u_1(zh_t - g_t, 1 - h_t)z(1 - \tau_t) = u_2(zh_t - g_t, 1 - h_t).$$

Only in exceptional cases (e.g., $u(.,.)$ is separable and logarithmic in both arguments) would the solution to this equation not depend on τ_t.

8. The production function for this economy is given by

$$Y_t = G_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1. \tag{PRF}$$

The preferences of the representative household are

$$E_0 \left(\sum\nolimits_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\gamma} C_t^{1-\gamma} (1 - L_t) \right] \right),$$
$$0 < \beta < 1, \quad 0 < \gamma < 1.$$

The capital accumulation equation is

$$K_{t+1} = (1+r) K_t + G_t^{\alpha} L_t^{1-\alpha} - C_t - G_t, \tag{CA}$$

with government spending following an AR(1) process around the log of its steady state value:

$$\widehat{g}_t \equiv \ln(G_t/G_{ss}) = \phi \widehat{g}_{t-1} + \varepsilon_t, \quad 0 \le \phi < 1, \tag{TS}$$

where $\{\varepsilon_t\}$ is an exogenous i.i.d. process.

(a) In recursive form, the social planner's problem is

$$V(K_t, G_t) = \max_{\{C_t, L_t\}} \frac{1}{1-\gamma} C_t^{1-\gamma} (1 - L_t) + \beta E_t \left(V \left((1+r) K_t + G_t^{\alpha} L_t^{1-\alpha} - C_t - G_t, G_{t+1} \right) \right).$$

The first order conditions are

$$C_t^{-\gamma} (1 - L_t) = \beta E_t \left(\frac{\partial V(K_{t+1}, G_{t+1})}{\partial K_{t+1}} \right), \tag{FOC1}$$

$$\frac{1}{1-\gamma} C_t^{1-\gamma} = \beta E_t \left(\frac{\partial V(K_{t+1}, G_{t+1})}{\partial K_{t+1}} \right) (1-\alpha) G_t^{\alpha} L_t^{-\alpha}. \tag{FOC2}$$

Using Benveniste and Scheinkman's results, we find that

$$\frac{\partial V(K_t, G_t)}{\partial K_t} = \beta E_t \left(\frac{\partial V(K_{t+1}, G_{t+1})}{\partial K_{t+1}} \right) (1+r).$$

Inserting equation (FOC1), this reduces to

$$\frac{\partial V(K_t, G_t)}{\partial K_t} = C_t^{-\gamma} (1 - L_t)(1+r),$$

and (FOC1) becomes

$$C_t^{-\gamma} (1 - L_t) = \beta (1+r) E_t \left(C_{t+1}^{-\gamma} (1 - L_{t+1}) \right) = E_t \left(C_{t+1}^{-\gamma} (1 - L_{t+1}) \right). \tag{EE}$$

Combining equations (FOC1) and (FOC2) yields

$$\frac{1}{1-\gamma} C_t^{1-\gamma} = C_t^{-\gamma} (1 - L_t)(1-\alpha) G_t^{\alpha} L_t^{-\alpha}$$

$$\frac{L_t^{\alpha}}{1 - L_t} = \frac{1}{C_t} (1-\gamma)(1-\alpha) G_t^{\alpha}, \tag{LL}$$

The capital accumulation equation (CA) was derived above when formulating the social planner's problem.

(b) Log both sides of equation (LL):

$$-\ln\left(1-\exp\left(\ln L_t\right)\right)+\alpha\ln\left(L_t\right)=\ln\left(\left(1-\alpha\right)\left(1-\gamma\right)\right)-\ln\left(C_t\right)+\alpha\ln\left(G_t\right).$$

Implicitly differentiating this expression yields

$$\alpha d\ln L_t-\frac{-\exp\left(\ln L_t\right)}{1-\exp\left(\ln L_t\right)}d\ln L_t=-d\ln C_t+\alpha d\ln G_t.$$

Letting lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values, this implies

$$\begin{aligned}\widehat{\ell}_t &\approx \left[\alpha+\frac{L_{ss}}{1-L_{ss}}\right]^{-1}\left(-\widehat{c}_t+\alpha\widehat{g}_t\right)\\ &\equiv \theta\left[\alpha\widehat{g}_t-\widehat{c}_t\right],\quad \theta>0, \qquad\qquad (\text{LL}')\end{aligned}$$

with the last line following from α, $L_{ss}\in(0,1)$. Proceeding similarly, it follows from equation (PRF) that

$$\begin{aligned}\widehat{y}_t &= \left(1-\alpha\right)\widehat{\ell}_t+\alpha\widehat{g}_t\\ &\approx \left[1+\left(1-\alpha\right)\theta\right]\alpha\widehat{g}_t-\left(1-\alpha\right)\theta\widehat{c}_t,\\ &\equiv \alpha(1+\lambda)\widehat{g}_t-\lambda\widehat{c}_t,\quad 0<\lambda<\theta. \qquad\qquad (\text{PRF}')\end{aligned}$$

(c) Implicitly differentiate equation (CA):

$$\begin{aligned}\exp\left(\ln\left(K_{t+1}\right)\right) &= \left(1+r\right)\exp\left(\ln\left(K_t\right)\right)+\exp\left(\ln\left(Y_t\right)\right)-\exp\left(\ln\left(C_t\right)\right)\\ &\quad -\exp\left(\ln\left(G_t\right)\right),\\ \exp\left(\ln\left(K_{t+1}\right)\right)d\ln\left(K_{t+1}\right) &= \left(1+r\right)\exp\left(\ln\left(K_t\right)\right)d\ln\left(K_t\right)+\exp\left(\ln\left(Y_t\right)\right)d\ln\left(Y_t\right)\\ &\quad -\exp\left(\ln\left(C_t\right)\right)d\ln\left(C_t\right)-\exp\left(\ln\left(G_t\right)\right)d\ln\left(G_t\right),\end{aligned}$$

so that

$$\begin{aligned}K_{ss}\widehat{k}_{t+1} &\approx \left(1+r\right)K_{ss}\widehat{k}_t+Y_{ss}\widehat{y}_t-C_{ss}\widehat{c}_t-G_{ss}\widehat{c}_t,\\ \widehat{k}_{t+1} &\approx \left(1+r\right)\widehat{k}_t+\frac{Y}{K}\widehat{y}_t-\frac{C}{K}\widehat{c}_t-\frac{G}{K}\widehat{g}_t\\ &= \left(1+r\right)\widehat{k}_t+\left(\psi+\zeta-r\right)\widehat{y}_t-\psi\widehat{c}_t-\zeta\widehat{g}_t.\end{aligned}$$

Then insert (PRF″) to get

$$\widehat{k}_{t+1}\approx\left(1+r\right)\widehat{k}_t+\left(\psi+\zeta-r\right)\left[\alpha\left(1+\lambda\right)\widehat{g}_t-\lambda\widehat{c}_t\right]-\psi\widehat{c}_t-\zeta\widehat{g}_t,$$

so that the log-linear approximation for the capital accumulation equation is

$$\begin{aligned}\widehat{k}_{t+1} &= \left(1+r\right)\widehat{k}_t+\omega_1\widehat{g}_t-\omega_2\widehat{c}_t, \qquad\qquad (\text{CA}')\\ \omega_1 &= \left(\psi+\zeta-r\right)\alpha\left(1+\lambda\right)-\zeta,\\ \omega_2 &= \left(\psi+\zeta-r\right)\lambda+\psi.\end{aligned}$$

Since $(\psi+\zeta)>r$, $\lambda>0$ and $\psi>0$, it follows that $\omega_2>0$. The sign of ω_1, however, is uncertain.

(d) It is straightforward to log-linearize the Euler equation, and show that:

$$\widehat{c}_t = \eta\widehat{k}_t + \mu\widehat{g}_t,$$

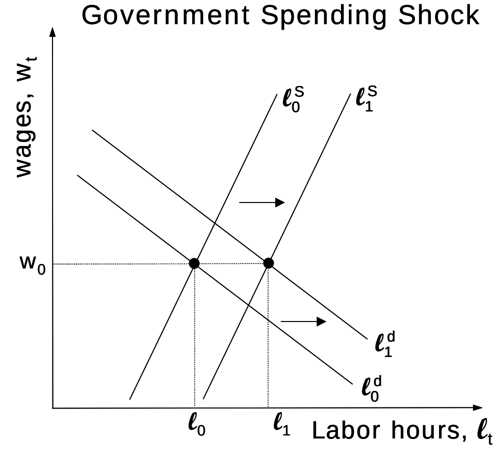
(This is given in the question.)

1. Combining equations (LL′), (PRF′) and (CF), we get

$$\begin{aligned}
\widehat{y}_t &= \alpha(1+\lambda)\widehat{g}_t - \lambda\left(\eta\widehat{k}_t + \mu\widehat{g}_t\right) \\
&= \left[\alpha + \lambda\left(\alpha - \mu\right)\right]\widehat{g}_t - \lambda\eta\widehat{k}_t, \\
\widehat{\ell}_t &= \theta\left[\alpha\widehat{g}_t - \left(\eta\widehat{k}_t + \mu\widehat{g}_t\right)\right] \\
&= \theta\left(\alpha - \mu\right)\widehat{g}_t - \theta\eta\widehat{k}_t, \\
\widehat{apl}_t &= \widehat{y}_t - \widehat{\ell}_t \\
&= \left[\alpha + \left(\lambda - \theta\right)\left(\alpha - \mu\right)\right]\widehat{g}_t - \left(\lambda - \theta\right)\eta\widehat{k}_t.
\end{aligned}$$

2. Given our assumption that $\mu < \alpha$, an unexpected government spending increase will increase both output and labor. However, with $\theta > \lambda$ the effect on the average product of labor is uncertain. When government spending goes up, two things occur. The first is that the spending itself crowds out private consumption, increasing labor supply and output. In this case, the diminishing marginal product of labor implies that $\widehat{apl}_t$ will decrease. Graphically, this appears as a rightward shift of the labor supply curve. The second effect is that government spending acts as a productivity increase, raising output, labor and $\widehat{apl}_t$. Graphically, this appears as a rightward shift of the labor demand curve. Combining these effects means output and hours will rise, but the effect on the average product of labor is uncertain. Extrapolating from the uncertain initial effects means $\widehat{apl}_t$ can be either pro- or counter-cyclical.

Initial Responses to a Productive Government Spending Shock



(e) Attached are the background derivations—**not** needed for the answers above—behind the shortcuts provided in the exam.

We begin by logging both sides of Equation (EE):

$$\begin{aligned} -\gamma \ln C_t + \ln\left(1 - \exp\left(\ln L_t\right)\right) &= \ln\left(E_t\left(C_{t+1}^{-\gamma}\left(1 - L_{t+1}\right)\right)\right) \\ &\approx E_t\left(-\gamma \ln C_{t+1} + \ln\left(1 - \exp\left(\ln L_{t+1}\right)\right)\right), \end{aligned}$$

with the second line using the approximation $\ln\left(E_t\left(X_t\right)\right) \approx E_t\left(\ln\left(X_t\right)\right)$. Implicitly differentiating, we get

$$-\gamma d \ln C_t + \frac{-\exp\left(\ln L_t\right)}{1 - \exp\left(\ln L_t\right)} d \ln L_t \approx E_t\left(-\gamma d \ln C_{t+1} + \frac{-\exp\left(\ln L_{t+1}\right)}{1 - \exp\left(\ln L_{t+1}\right)} d \ln L_{t+1}\right),$$

or

$$\gamma \widehat{c}_t + \frac{L_{ss}}{1 - L_{ss}} \widehat{\ell}_t \approx E_t\left(\gamma \widehat{c}_{t+1} + \frac{L_{ss}}{1 - L_{ss}} \widehat{\ell}_{t+1}\right).$$

Inserting equation (LL′), we get

$$\gamma \widehat{c}_t + \frac{L_{ss}}{1 - L_{ss}} \theta\left[\alpha \widehat{g}_t - \widehat{c}_t\right] \approx E_t\left(\gamma \widehat{c}_{t+1} + \frac{L_{ss}}{1 - L_{ss}} \theta\left[\alpha \widehat{g}_{t+1} - \widehat{c}_{t+1}\right]\right).$$

Note that

$$\omega_4 \equiv \frac{L_{ss}}{1 - L_{ss}} \theta = \frac{L_{ss}}{1 - L_{ss}}\left[\alpha + \frac{L_{ss}}{1 - L_{ss}}\right]^{-1} \in (0, 1).$$

Using this notation, we have

$$\left(\gamma - \omega_4\right) \widehat{c}_t + \omega_4 \alpha \widehat{g}_t \approx E_t\left(\left(\gamma - \omega_4\right) \widehat{c}_{t+1} + \omega_4 \alpha \widehat{g}_{t+1}\right),$$

which can be written as

$$\begin{aligned} \widehat{c}_t + \omega_3 \widehat{g}_t &\approx E_t\left(\widehat{c}_{t+1} + \omega_3 \widehat{g}_{t+1}\right), && \text{(EE}'\text{)} \\ \omega_3 &= \frac{\omega_4}{\gamma - \omega_4} \alpha = \alpha \frac{1}{\gamma / \omega_4 - 1} \end{aligned}$$

Next, let's derive μ and η. Combining equations (CF) and equation (CA′) yields

$$\widehat{k}_{t+1} = \left(1 + r - \omega_2 \eta\right) \widehat{k}_t + \left(\omega_1 - \omega_2 \mu\right) \widehat{g}_t. \qquad \text{(CA}''\text{)}$$

Inserting equation (CF) into the *right*-hand-side of equation (EE′) yields

$$E_t\left(\widehat{c}_{t+1} + \omega_3 \widehat{g}_{t+1}\right) = \eta \widehat{k}_t + \mu \widehat{g}_t + \omega_3 \widehat{g}_t,$$

Inserting equation (CF) into the *left*-hand-side of equation (EE′) yields

$$E_t\left(\widehat{c}_{t+1} + \omega_3 \widehat{g}_{t+1}\right) = E_t\left(\eta \widehat{k}_{t+1} + \mu \widehat{g}_{t+1} + \omega_3 \widehat{g}_{t+1}\right).$$

Inserting (CA″) into this expression and recalling equation (TS) yields

$$\begin{aligned} E_t\left(\widehat{c}_{t+1} + \omega_3 \widehat{g}_{t+1}\right) &= E_t\left(\eta\left[\left(1 + r - \omega_2 \eta\right) \widehat{k}_t + \left(\omega_1 - \omega_2 \mu\right) \widehat{g}_t\right] + \mu \widehat{g}_{t+1} + \omega_3 \widehat{g}_{t+1}\right), \\ &= \eta\left(1 + r - \omega_2 \eta\right) \widehat{k}_t + \left(\eta\left[\omega_1 - \omega_2 \mu\right] + \phi\left[\mu + \omega_3\right]\right) \widehat{g}_t. \end{aligned}$$

If the expressions for $E_t\left(\widehat{c}_{t+1}\right)$ are to be equal,

$$\begin{aligned}\eta\left(1+r-\omega_2\eta\right) &= \eta, \\ \eta[\omega_1-\omega_2\mu]+\phi\left[\mu+\omega_3\right] &= \mu+\omega_3.\end{aligned}$$

The first of these two equations has the non-zero solution

$$1+r-\omega_2\eta=1 \Rightarrow \eta=\frac{r}{\omega_2},$$

while the second equation implies that

$$\begin{aligned}\eta\omega_1-(1-\phi)\,\omega_3 &= \mu\left(1-\phi+\eta\omega_2\right) \\ &= \mu\left(1-\phi+r\right),\end{aligned}$$

so that

$$\begin{aligned}\mu &= \frac{\eta\omega_1}{1-\phi+r}-\frac{(1-\phi)\,\omega_3}{1-\phi+r} \\ &= \frac{r\omega_1}{\omega_2\left(1-\phi+r\right)}-\frac{(1-\phi)\,\omega_3}{1-\phi+r} \\ &= \frac{r\omega_1}{\omega_2\left(1-\phi+r\right)}-\alpha\frac{1}{\gamma/\omega_4-1}\left(\frac{1-\phi}{1-\phi+r}\right).\end{aligned}$$

Note that

$$\begin{aligned}\frac{\omega_1}{\omega_2} &= \frac{(\psi+\zeta-r)\,\alpha\,(1+\lambda)-\zeta}{(\psi+\zeta-r)\,\lambda+\psi} \\ &= \frac{(\psi+\zeta-r)\,\alpha\lambda+\alpha\psi-(1-\alpha)\,\zeta-\alpha r}{(\psi+\zeta-r)\,\lambda+\psi} \\ &< \frac{(\psi+\zeta-r)\,\alpha\lambda+\alpha\psi}{(\psi+\zeta-r)\,\lambda+\psi}=\alpha.\end{aligned}$$

Then with $\phi<1$ we have

$$\mu+\frac{(1-\phi)\,\omega_3}{1-\phi+r}=\frac{r\omega_1}{\omega_2\left(1-\phi+r\right)}<\frac{r}{1-\phi+r}\alpha<\alpha,$$

and for positive values of ω_3, we have $\mu<\alpha$. In general, however, the sign and size of μ are indeterminate, reflecting the multiple effects of government spending that were discussed above.

9. We are considering an economy populated by N Democrats and N Republicans. The preferences of both consumers are

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\sigma}\left[c_t^{1-\sigma}-1\right]\right),\quad 0<\beta<1,\quad \sigma>0.$$

Output is produced by an infinite-lived tree: each period, the tree produces d units of non-storable output. However, there is political uncertainty. When Democrats are in charge of the government, they force each Republican to transfer m units of output to Democrats. When Republicans are in charge, the situation reverses. The indicator $e_t \in \{D_t, R_t\}$, which denotes which party is in charge, follows a stationary, symmetric two-state Markov chain with stationary transition density $f(e', e)$:

$$\begin{aligned}
f(e,e) &= \Pr(e_{t+1}=D|\, e_t=D)=\pi\\
&= f(R,R),\\
f(D,R) &= \Pr(e_{t+1}=D|\, e_t=R)=1-\pi\\
&= f(R,D).
\end{aligned}$$

(a) Let $q(e', e)$ denote the price of the one-step-ahead contingent claim that pays off when $e_{t+1}=e'$. Writing the Republican's problem as a Lagrangean, we get

$$\begin{aligned}
&V(x_t,e_t)=\\
&\min_{\lambda_t\geq 0}\ \max_{c_t\geq 0,\ s_{t+1},\ z(E')}\ \frac{c_t^{1-\sigma}-1}{1-\sigma}+\lambda_t\left(x_t-c_t-p_ts_{t+1}-\sum_{e'\in\{D,R\}}q(e',e_t)\,z(e')\right)\\
&\quad+\beta\sum_{e_{t+1}\in\{D,R\}}f(e_{t+1},e_t)\times V\left(\left(\begin{array}{c} z(e_{t+1})+[p_{t+1}(e_{t+1})+d]\,s_{t+1}\\ +\,m\left[1-2\times I_{e_{t+1}=D}\right]\end{array}\right),\quad e_{t+1}\right),
\end{aligned}$$

The FOC for an interior solution are:

$$\begin{aligned}
c_t^{-\sigma} &= \lambda_t,\\
\lambda_t p_t &= \beta\sum_{e_{t+1}\in\{D,R\}}f(e_{t+1},e_t)\times\frac{\partial V[t+1]}{\partial x_{t+1}}\left[p_{t+1}(e_{t+1})+d\right],\\
\lambda_t q(e',e_t) &= \beta\frac{\partial V(x_{t+1}(e'),e')}{\partial x_{t+1}}f(e',e_t),\quad e'\in\{D,R\}.
\end{aligned}$$

Since (following Benveniste-Scheinkman),

$$\frac{\partial V[t]}{\partial x_t}=\lambda_t,$$

the Euler equations are

$$p_tc_t^{-\sigma} = \beta E_t\left(c_{t+1}(e_{t+1})^{-\sigma}\left[p_{t+1}(e_{t+1})+d\right]\right), \tag{EE1}$$

$$q(e',e_t) = \beta\left(\frac{c_t}{c_{t+1}(e')}\right)^{\sigma}f(e',e_t),\quad e'\in\{D,R\}. \tag{EE2}$$

(b) We consider first an arrangement where Democrats and Republicans refuse to trade with each other: Democrats will trade only with Democrats, etc.

1. Evaluating equation (EE2) shows that the equilibrium pricing kernel for Democrats is

$$\begin{aligned} q(D,D) &= \beta\left(\frac{d+m}{d+m}\right)^{\sigma}\pi=\beta\pi, \\ q(R,D) &= \beta\left(\frac{d+m}{d-m}\right)^{\sigma}(1-\pi), \\ q(D,R) &= \beta\left(\frac{d-m}{d+m}\right)^{\sigma}(1-\pi), \\ q(R,R) &= \beta\left(\frac{d-m}{d+m}\right)^{\sigma}\pi=\beta\pi. \end{aligned}$$

The kernel for Republicans is the mirror opposite:

$$\begin{aligned} q(D,D) &= \beta\pi, \\ q(R,D) &= \beta\left(\frac{d-m}{d+m}\right)^{\sigma}(1-\pi), \\ q(D,R) &= \beta\left(\frac{d+m}{d-m}\right)^{\sigma}(1-\pi), \\ q(R,R) &= \beta\pi. \end{aligned}$$

2. It follows from arbitrage arguments that if an asset pays $w(e)$ units of consumption goods when $e_{t+1}=e$, its price is

$$p_t^w=\int w(e_{t+1})\,q(e_{t+1},e)\,de_{t+1}.$$

When the asset is a risk-free discount bond, $w(e)=1$. Imposing the pricing kernel, it follows that the price of a risk-free bond, $R^{-1}(e_t)$, is given by

$$R^{-1}(e)=\sum_{e_{t+1}\in\{D,R\}} q(e_{t+1},e_t).$$

For Republicans we have

$$\begin{aligned} R^{-1}(R) &= q(D,R)+q(R,R)=\beta\left(\frac{d+m}{d-m}\right)^{\sigma}(1-\pi)+\beta\pi, \\ R^{-1}(D) &= q(D,D)+q(R,D)=\beta\pi+\beta\left(\frac{d-m}{d+m}\right)^{\sigma}(1-\pi). \end{aligned}$$

Again, everything reverses for Democrats:

$$\begin{aligned} R^{-1}(R) &= q(D,R)+q(R,R)=\beta\left(\frac{d-m}{d+m}\right)^{\sigma}(1-\pi)+\beta\pi, \\ R^{-1}(D) &= q(D,D)+q(R,D)=\beta\pi+\beta\left(\frac{d+m}{d-m}\right)^{\sigma}(1-\pi). \end{aligned}$$

(c) If a financial intermediary enters the economy and enables financial markets to be complete, consumers will take out contingent claims that eliminate all idiosyncratic political risk. Because there is no aggregate risk, the end result is that both types of consumers will consume the same amount, d, every period.

1. In equilibrium, with $c_t = d$, equation (EE2) simplifies to

$$\begin{aligned} q(D,D) &= \beta\pi = q(R,R), \\ q(R,D) &= \beta(1-\pi) = q(D,R), \end{aligned}$$

for both types of consumers.

2. The equilibrium pricing function for bonds, $R^{-1}(E_t)$, becomes

$$R^{-1}(D) = q(D,D) + q(R,D) = \beta.$$

3. In equilibrium, with $c_t = d$, equation (EE1) simplifies to

$$p_t = \beta E_t([p_{t+1}(e_{t+1}) + d])$$

Since $0 < \beta < 1$, it makes sense to solve this equation forward:

$$E_t\left(\left(1-\beta L^{-1}\right)p_t\right) = \beta E_t(d),$$

so that

$$\begin{aligned} p_t &= \frac{1}{1-\beta L^{-1}}\beta d + b_t \\ &= \beta d \sum_{t=0}^{\infty} \beta^t + b_t, \end{aligned}$$

with the "bubble term" b_t obeying

$$E_t(b_{t+1}) = \beta^{-1} b_t.$$

We now require that

$$\lim_{J\to\infty} E_t\left(\beta^J p_{t+J}\right) = 0, \quad \forall t. \tag{TVC}$$

Equation (TVC) will be satisfied only if $b_t = 0$ and the price of a stock is

$$p_t = \frac{\beta}{1-\beta} d.$$

4. With no aggregate risk, there is no uncertainty about the return on stocks, and the equity premium is zero.

10. **Search with on-the-job wage changes.** (Adapted from Rogerson, Shimer & Wright.)

- **Time:** Discrete, infinite horizon.
- **Demography:** Single worker who lives for ever.
- **Preferences:** The worker is risk-neutral (i.e. $u(x) = x$) and discounts the future at the rate r.
- **Endowments:** *When unemployed:* The worker receives b units of the consumption good per period. Also, with probability α she gets to sample a wage from the continuous distribution F with support $(0, \bar{w}]$ where $\bar{w} > b$.
- *When employed*: The worker receives her current wage but the wage can change. There is no lay-off as such but instead with probability λ a new wage is drawn from F. The worker can quit if she considers the new wage to be too low. Otherwise, she remains employed at the new wage and is again subject to the same probability of a new draw. (Note: the worker does not have the option of remaining in the job at her old wage.)

(a) The flow asset value equations for V_u (the value to being unemployed) and $V_e(w)$ (the value to being employed at the wage w) are

$$\begin{aligned} rV_u &= b + \alpha \mathbb{E}_w \left[\max\{V_e(w) - V_u, 0\}\right] \\ rV_e(w) &= w + \lambda \mathbb{E}_{w'} \left[\max\{V_e(w') - V_e(w), V_u - V_e(w)\}\right] \\ &= w + \lambda \mathbb{E}_{w'} \left[\max\{V_e(w') - V_u, 0\}\right] + \lambda(V_u - V_e(w)). \end{aligned}$$

(b) The reservation wage w^* for an unemployed worker considering a new job offer solves $V_e(w^*) = V_u$. As an employed worker does not have the option of holding onto his old wage, his choice is always between continued employment at his current wage and unemployment. In short, continued employment at the wage w is the same as starting a new job at the wage w. This means that the reservation wage w^* for an employed worker considering whether to quit also solves $V_e(w^*) = V_u$, so that the reservation wage for unemployed workers is the same as the threshold wage below which employed workers will quit.

(c) From the equations above and the definition of mathematical expectation we have

$$\begin{aligned} rV_u &= b + \alpha \int_{w^*}^{\bar{w}} (V_e(w) - V_u)\, dF(w), \\ rV_e(w) &= w + \lambda \int_{w^*}^{\bar{w}} (V_e(w') - V_u)\, dF(w') + \lambda(V_u - V_e(w)). \end{aligned}$$

Subtracting yields

$$r\left(V_e(w) - V_u\right) = w - b + (\lambda - \alpha) \int_{w^*}^{\bar{w}} (V_e(w') - V_u)\, dF(w') + \lambda(V_u - V_e(w)),$$

so that

$$(r + \lambda)\left(V_e(w) - V_u\right) = w - b + (\lambda - \alpha) \int_{w^*}^{\bar{w}} (V_e(w') - V_u)\, dF(w'). \qquad (1)$$

Moreover, since $V_e(w^*) = V_u$ evaluating (1) at $w = w^*$ implies

$$0 = w^* - b + (\lambda - \alpha) \int_{w^*}^{\bar{w}} (V_e(w') - V_u)\, dF(w'). \tag{2}$$

Inserting (2) into (1) to eliminate the integral term implies

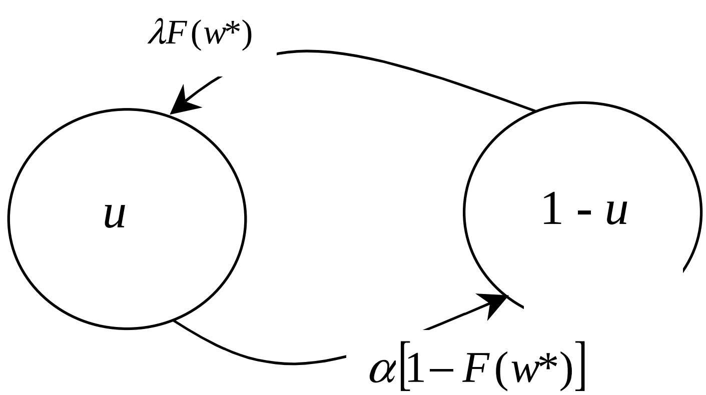
$$(r + \lambda)\,(V_e(w) - V_u) = w - w^*.$$

Substituting this expressing back into (2) and rearranging, we get

$$w^* = b + \frac{\alpha - \lambda}{r + \lambda} \int_{w^*}^{\bar{w}} (w' - w^*)\, dF(w').$$

(d) The preceding answer shows that if $\lambda = \alpha$, $w^* = b$. If there is no opportunity cost (a smaller flow of wage offers) associated with accepting a job then I accept any offer that is better than my current income (b).

(e) If there is a large number of such workers with mass normalized to 1, the steady-state flow diagram for movements between employment and unemployment is given by



Letting u denote the fraction of workers that are unemployed, we see that in a steady state

$$\alpha[1 - F(w^*)]u = \lambda F(w^*)(1 - u),$$

so that

$$u = \frac{\lambda F(w^*)}{\alpha[1 - F(w^*)] + \lambda F(w^*)}.$$