

Macroeconomics

Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.*

1. If consumers receive utility from government goods, then government spending has no wealth effects.
2. A higher saving/income ratio causes faster long-run economic growth.
3. Risk averse investors will always prefer assets that have a known rate of return.
4. Real business cycle theory accurately describes the current macroeconomic situation: the oil price is up, auto sales are down, exports are booming, house prices are falling, and a credit crunch has whacked the banks.
5. Firms that have constant returns to scale technologies and take factor prices as given cannot make positive profits.
6. The high expected rate of return to investors explains the low personal saving rate during the 1990's stock boom and the 2000's housing boom.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Proportional income taxes in simple dynamic model with a government.

Consider the following economy:

- **Time:** Discrete; infinite horizon
- **Demography:** Continuum of mass 1 of (representative) consumer/worker households, and a large number of profit-maximizing firms, owned jointly by the households.
- **Preferences:** The instantaneous household utility function is $u(c)$ where c is household consumption and $u(\cdot)$ is strictly increasing and strictly concave. The discount factor is $\beta \in (0, 1)$.
- **Technology:** There is a constant returns to scale technology for which labor is the only input so that a firm that hires h units of labor produces zh units of output.
- **Endowments:** Each household has 1 unit of time per period to allocate between work and leisure.
- **Institutions:** There is a government that has to meet an exogenous stream of expenditures, $\{g_t\}$. Government spending is thrown into the ocean. The government can levy taxes and issue bonds in order to meet its expenditure requirement. Taxes are restricted to being proportional to labor income so that in period t , the tax revenue from a household which provides labor services h_t is $\tau_t w_t h_t$, where τ_t is the period t tax rate and w_t is the wage rate. Every period there are markets for labor, government bonds and consumption goods.

- (a) Write down and solve the problems faced by the representative household and the representative firm.
- (b) Write down the government's flow (period-by-period) budget constraint.
- (c) Define and characterize a competitive equilibrium.
- (d) Does Ricardian equivalence hold? Explain.
- (e) How would your answer to part (d) change if the utility function was replaced by $u(c_t, 1 - h_t)$ and $u(\cdot, \cdot)$ is strictly increasing in both arguments? Explain your answer.

8. Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,$$

$$Y_t = G_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (\text{PRF})$$

where: Π_t is profit; L_t is labor; W_t is the real wage; Y_t is output; and G_t is the government's purchases of goods and services.

The preferences of the representative household over consumption, C_t , and labor are given by

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\gamma} C_t^{1-\gamma} (1-L_t) \right] \right),$$

$$0 < \beta < 1, \quad 0 < \gamma < 1.$$

Households receive labor income and profits from firms. Households earn a gross return of $(1+r)K_t$ on their assets, K_t , with $\beta(1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1} , are chosen in period t . Households also face the usual non-negativity and No-Ponzi-Game conditions.

The government's flow budget constraint is given by

$$H_t = G_t,$$

where H_t is the total value of lump-sum taxes. Note that lump-sum taxes are driven by government spending, which follows an AR(1) process around the log of its steady state value:

$$\hat{g}_t \equiv \ln(G_t/G_{ss}) = \phi \hat{g}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \quad (\text{TS})$$

where $\{\varepsilon_t\}$ is an exogenous i.i.d. process, and G_{ss} is steady state government spending. (You can also assume that $G_t < Y_t$, $\forall t$.)

- (a) Write down the social planner's problem for this economy as a dynamic programming problem. (The planner takes G_t as given.) Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation.
- (b) Let lower-case letters with carats " $\hat{\cdot}$ " denote deviations of logged variables around their steady state values. Show that log-linearized expressions for labor and output are

$$\begin{aligned} \hat{\ell}_t &= \theta [\alpha \hat{g}_t - \hat{c}_t], \\ \hat{y}_t &= \alpha(1+\lambda) \hat{g}_t - \lambda \hat{c}_t, \quad \theta > \lambda > 0. \end{aligned} \quad (\text{LM})$$

- (c) Suppose that the steady state consumption-to-capital ratio, C/K , is ψ , and that the steady state government spending-to-capital ratio is ζ , with $(\psi + \zeta) > r$. It is then straightforward to show that the steady state output-to-capital ratio, Y/K , is $\psi + \zeta - r$. (Take this as given.) Using this result, log-linearize the capital accumulation equation to show that

$$\hat{k}_{t+1} = (1+r) \hat{k}_t + \omega_1 \hat{g}_t - \omega_2 \hat{c}_t.$$

- (d) One can log-linearize the Euler equation and solve the resulting system to express consumption as a function of capital and government spending:

$$\widehat{c}_t = \eta \widehat{k}_t + \mu \widehat{g}_t,$$

with $\eta \in (0, 1)$. (Take this as given.) Consider the log of average labor productivity, $\widehat{apl}_t = \widehat{y}_t - \widehat{\ell}_t$.

1. Express $\widehat{\ell}_t$, \widehat{y}_t and \widehat{apl}_t as functions of government spending and capital.
 2. Suppose that the economy experiences an unexpected government spending increase (\widehat{g} rises). Assuming that $\mu < \alpha$, in what direction will output, labor and average productivity initially respond? Extrapolating from the initial response, would you expect average labor productivity to be pro- or counter-cyclical in this model? Explain intuitively, including graphical analysis.
9. Consider the following variant of the Lucas tree model. The economy is populated by N Democrats and N Republicans, where N is a large number. The preferences of both consumers are

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [c_t^{1-\sigma} - 1] \right), \quad 0 < \beta < 1, \quad \sigma > 0.$$

where c_t denotes consumption, and $E_t(\cdot)$ denotes expectations conditional on time- t information.

Output is produced by an infinite-lived tree: each period, the tree produces d units of non-storable output.

Uncertainty in this economy is driven by politics. When Democrats are in charge of the government, they force each Republican to transfer $m \in (0, d)$ units of output to Democrats. When Republicans are in charge, the situation reverses, and each Democrat must transfer m units of output to Republicans. Let the indicator $e_t \in \{D_t, R_t\}$ denote which party is in charge. Assume that e_t follows a symmetric two-state Markov chain with the stationary transition density $f(e', e)$:

$$\begin{aligned} f(D, D) &= \Pr(e_{t+1} = D | e_t = D) = \pi \\ &= f(R, R), \\ f(D, R) &= \Pr(e_{t+1} = D | e_t = R) = 1 - \pi \\ &= f(R, D). \end{aligned}$$

Let $p_t = p(e_t)$ be the price at time t of a title to all future dividends from a tree, and let $q(e', e_t)$ be the kernel used to price the one-step-ahead contingent claim that delivers one unit of the consumption good when $e_{t+1} = e'$. Finally, let x_t denote the consumer's financial resources, which she allocates between stocks, contingent claims and consumption. The economy starts off with each consumer owning one tree, and a one-time occurrence of political detente: at period 0, there are no transfers.

- (a) Write down the problem for one type of consumer—either a Democrat or a Republican—in recursive form, and find the first order conditions. (Notation **Hint**: let $I_{\mathcal{A}}$ denote the 0-1 indicator function that returns 1 when event \mathcal{A} occurs.)

- (b) Consider first an arrangement where Democrats and Republicans refuse to trade with each other: Democrats will trade only with Democrats, and Republicans will trade only with Republicans.
1. Find the equilibrium pricing kernel, $q(e', e_t)$, first for Democrats and then for Republicans, for all (e', e_t) combinations.
 2. Let $R_t^{-1} = R_t^{-1}(e_t)$ be the time- t price of a risk-free discount bond that pays one unit of consumption at time $t + 1$ under any state. Find the equilibrium pricing function for bonds, for both types of consumers.
- (c) Now suppose a neutral financial intermediary enters the economy, willing to trade with anyone. Because Democrats and Republicans no longer have to interact directly to trade, this allows financial markets to be complete.
1. Find the equilibrium pricing kernel, $q(e', e_t)$, for both types of consumers.
 2. Find the equilibrium pricing function for bonds, $R^{-1}(e_t)$, for both types of consumers.
 3. Find the price of stock, $p(e_t)$, for both types of consumers.
 4. Find the equity premium. You do not need to actually do the calculations—a succinct argument will suffice.

10. **Search with on-the-job wage changes.** (Adapted from Rogerson, Shimer & Wright.)

- **Time:** Discrete, infinite horizon.
 - **Demography:** Single worker who lives for ever.
 - **Preferences:** The worker is risk-neutral (i.e. $u(x) = x$) and discounts the future at the rate r .
 - **Endowments:** *When unemployed:* The worker receives b units of the consumption good per period. Also, with probability α she gets to sample a wage from the continuous distribution F with support $(0, \bar{w}]$ where $\bar{w} > b$.
 - *When employed:* The worker receives her current wage but the wage can change. There is no lay-off as such but instead with probability λ a new wage is drawn from F . The worker can quit if she considers the new wage to be too low. Otherwise, she remains employed at the new wage and is again subject to the same probability of a new draw. (Note: the worker does not have the option of remaining in the job at her old wage.)
- (a) Obtain the flow asset value equations for V_u (the value to being unemployed) and $V_e(w)$ (the value to being employed at the wage w).
 - (b) Explain how these asset value equations imply that the reservation wage, w^* for unemployed workers is the same as the threshold wage below which employed workers will quit - provide intuition.
 - (c) Derive the reservation wage equation. (Hint: Obtain an expression for $V_e(w) - V_u$, then evaluate it at $w = w^*$ and substitute it back in to the expression.)
 - (d) If $\lambda = \alpha$, how does the reservation wage compare to b ? What is the intuition for this result?
 - (e) Assuming there is a large number of such workers with mass normalized to 1, construct the steady-state flow diagram for movements between employment and unemployment. Use this to obtain an expression for the steady-state unemployment rate, u , in terms of model parameters and w^* .