Macroeconomics

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements*, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.

1. Most of the expenditures in President Obama's economic stimulus plan will be made some time in the future. As a result, the plan will have almost no effect on the economy today.
2. Sticky-price Neo-Keynesian models show that the IS-LM model is fully consistent with rational economic behavior.
3. Putting money in the utility function is never a valid modeling technique in an infinite horizon environment.
4. Search is irrelevant for modelling retail markets: "when I want shoes I go to a shoe store; when I want coffee I go to a coffee shop".
5. In either the Keynesian macroeconomic model or the real business cycle model, banks play no role. Each consumer maximizes utility subject to his budget constraint, and each firm maximizes profit. Banks and other financial intermediaries channel funds from savers to investors, but this activity is invisible in each model.
6. Aggregate consumption demand responds to stock price fluctuations in accord with economic theory. A drop in the stock market reduces the wealth of investors, and each investor reduces his consumption in line with how the fall in wealth tightens his budget constraint. Consequently the ups and downs of the stock market are the key to understanding the business cycle

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. (Inspired by Edge, Laubach and Williams, 2007.) Consider the following simplified version of a stochastic growth model, where technology is observed with a lag. The representative price-taking firm solves

$$\max_{L_t \geq 0} E\left(\Pi_t \middle| I_t\right) = E\left(Y_t \middle| I_t\right) - W_t L_t,$$

$$Y_t = A_t L_t^{\eta}, \quad 0 < \eta < 1, \tag{PRF}$$

where: Y_t is output; L_t is labor; W_t is the real wage; A_t measures the firm's productivity; and $E_t(\cdot) = E(\cdot| I_t)$ denotes the conditional expectation operator.

The representative household solves the following problem:

$$\max_{\{C_t, B_{t+1}, L_t\}_{t=0}^{\infty}} E\left(\sum\nolimits_{t=0}^{\infty} \beta^t \left[C_t - \chi \frac{1}{1+\gamma} L_t^{1+\gamma}\right] \middle| I_0\right),$$

$$\gamma \geq 0, \quad \chi > 0, \quad 0 < \beta < 1,$$

$$s.t. \quad C_t + B_{t+1} = (1 + r_t) B_t + W_t L_t + \Pi_t, \tag{FBC}$$

and the usual boundary conditions. C_t is consumption and B_t is the consumer's holdings of risk-free bonds. Note that there is no storage technology; the risk-free bonds represent trades between consumers.

The log of productivity follows an AR(1) process

$$a_t \equiv \ln(A_t) = \phi a_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \tag{TS}$$

where $\{\varepsilon_t\}$ is an i.i.d. shock. However, at time t households and firms only observe $Z_t = A_t U_t$, where $u_t \equiv \ln(U_t)$ is i.i.d. measurement error. Assume further that

$$\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2); \quad u_t \sim \mathcal{N}(0, \sigma_u^2); \quad cov(\varepsilon_t, u_s) = 0, \quad \forall t, s.$$

Agents do observe lagged values of A, however, so that the time-t information set is

$$I_t = \{W_t, r_t, B_t, Z_t, A_{t-1}\},$$

which implies that the firm predicts A_t (and picks L_t) on the basis of Z_t and A_{t-1}. Note that the household must pick C_t before profits, Π_t, are realized.

(a) Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the resource constraint. (**Hint:** although the changes in the information structure will affect your derivation, the first order conditions will otherwise be standard.)

(b) Define $\widetilde{a}_t = E(a_t | I_t)$. It can be shown that $\widetilde{a}_t$ is

$$\widetilde{a}_t = \lambda(\varepsilon_t + u_t) + \phi a_{t-1}, \quad 0 < \lambda < 1, \tag{PRJ}$$

(take this as given) and that v_t, the forecast error for a_t, is

$$v_t \equiv a_t - \widetilde{a}_t = (1-\lambda)\,\varepsilon_t - \lambda u_t.$$

Let lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values. Show that log-linear approximations for labor and output are

$$\widehat{\ell}_t = \theta \widetilde{a}_t, \quad \theta > 0, \qquad \text{(LM)}$$
$$\widehat{y}_t = (1+\mu)\,a_t - \mu\left[(1-\lambda)\,\varepsilon_t - \lambda u_t\right], \quad \mu > 0. \qquad \text{(PRF}'\text{)}$$

Hint: at low levels of variance,

$$\ln\left(E\left(A_t|\, I_t\right)\right) \approx E\left(\ln(A_t)|\, I_t\right).$$

(c) Let's consider the economy's response to a technology shock.

1. Suppose that $\phi = 0.9$, $\gamma = 1/3$, $\eta = 2/3$ and $\lambda = 1/2$. Show that

$$\widehat{y}_t = 2a_t - \frac{1}{2}\varepsilon_t + \frac{1}{2}u_t. \qquad \text{(PRF}''\text{)}$$

2. Suppose further that: $\widehat{y}_0 = a_0 = \varepsilon_0 = 0$; $u_0 = u_1 = u_2..... = 0$; $\varepsilon_1 = 1$; $\varepsilon_2 = \varepsilon_3 = ... = 0$. Using your answer to part (1), trace out the effects of this shock by filling in the following table.

t	ε_t	a_t	u_t	$\widehat{y}_t$
0	0.00	0.00	0.00	0.00
1	1.00		0.00	
2	0.00		0.00	
3	0.00		0.00	
4	0.00		0.00	

3. Does the impulse response function for output resemble the impulse response function observed in the data? Briefly explain the mechanics that make it match or not match.

8. **Overlapping generations with acquired human capital.**

Consider the following economy:

Time: discrete, infinite horizon

Demography: A mass $N_t \equiv N_0(1+n)^t$ of newborns enter in every period. Everyone lives for 2 periods except for the first generation of old people.

Preferences: for the generations born in and after period 0:

$$U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \beta u(c_{2,t+1}),$$

where $c_{i,t}$ is consumption in period t and stage i of life, $u(.)$ is increasing strictly concave and twice differentiable $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$. For the initial old generation $\tilde{U}(c_{2,0}) = u(c_{2,0})$.

Productive technology: $f(h,l)$ is a neoclassical production technology which uses human capital and labor to produce a consumption good, f is homogeneous of degree 1, twice differentiable, increasing in both arguments, strictly concave with $\lim_{h\to 0} f_1(h,l) = \infty$ for all $l > 0$ and $\lim_{l\to 0} f_2(h,l) = \infty$ for all $h > 0$.

Human Capital Technology: $h_{t+1} = 1 - l_t$. Time not spent working in youth contributes to human capital available in old age.

Endowments: Everyone has one unit of labor services when young. They can use any fraction of this to work in the first period of life. Any unused labor services become human capital for use in the second period of life. (When old, only human capital can be used to earn money for consumption.) The initial old have h_0 units of human capital.

Institutions: There are competitive markets, for labor and human capital. (You can think of a single collectively owned firm which takes wages and interest rates as given.) Using the consumption good as the numeraire, let the per unit wage for each market in period t be w_t^l and w_t^h for labor and human capital respectively.

(a) Write out and solve the problems faced by generation t workers and period t firms in this economy.

(b) Define a competitive equilibrium and solve for the implied law of motion for the per-old person stock of human capital, h_t, in the economy.

(c) If $u(c) = \ln c$ and $f(h,l) = Ah^{\alpha}l^{1-\alpha}$, what is the law of motion for h_t?

(d) Maintaining these functional forms, solve for the steady state level of human capital, $\bar{h}$. What can you say about its dynamic properties (i.e. stability, oscillatory)?

9. **Asset pricing with money.** Consider an economy with a representative consumer who maximizes

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\left[\ln(c_t)+\alpha\ln(m_t)\right]\right),\quad 0<\beta<1,\quad \alpha>0,$$

where c_t denotes consumption, m_t denotes real balances. and $E_t(\cdot)$ denotes expectations conditional on time-t information.

The sole source of the single non-storable good is an everlasting tree that produces d_t units of the consumption good in period t. At the beginning of time 0, each consumer owns one such tree. Dividends are exogenous and follow a time-invariant Markov process.

The consumer's wealth evolves according to

$$M_t+P_t(c_t+R_t^{-1}b_{t+1}+q_t s_{t+1})+I_t^{-1}N_t=P_t(\tau_t+[q_t+d_t]s_t+b_t)+N_{t-1}+M_{t-1},\quad \text{(NFBC)}$$

where:

M_t is the nominal money supply and P_t is the aggregate price level;

q_t is the real price at time t of a title to all future dividends from the tree, and s_t is the number of trees;

R_t^{-1} is the real price of a risk-free discount bond that pays one unit of consumption at time $t+1$, and b_t is the (real) quantity of such bonds;

I_t^{-1} is the nominal price of a discount bond that pays one unit of money at time $t+1$, and N_t is the (nominal) quantity of such bonds. Note that nominal bonds are dated in the same way as money, rather than stocks or real bonds.

The nominal money supply M_t evolves according to

$$M_t=(1+\theta_t)M_{t-1},\quad \text{(MS)}$$

with θ_t following an exogenous, time-invariant Markov process. In equilibrium, lump-sum taxes equal seigniorage:

$$\tau_t=\frac{M_t-M_{t-1}}{P_t}.\quad \text{(GBC)}$$

The consumer, however, takes τ_t as given.

(a) Let $\pi_t=P_t/P_{t-1}-1$ denote the rate of inflation and let $n_t=N_t/P_t$ denote the real quantity of nominal bonds. Convert the nominal flow budget constraint (NFBC) into a real flow budget constraint.

(b) Write down the consumer's problem in recursive form, and find the first order conditions.

(c) Find the equilibrium allocation of stocks, bonds and consumption. Find the equilibrium prices for real bonds, R_t^{-1}, and nominal bonds, I_t^{-1}.

(d) According to the Fisher equation, nominal and real interest rates are related by $I_t=I_{Ft}\equiv R_tE_t(1+\pi_{t+1})$.

1. Using your answers to part (c), find $e_t=I_t-I_{Ft}$. The deviation e_t should reflect a risk premium and an approximation error.
2. Interpret your findings.

10. **Cash-in-Advance with home and market goods**

Consider the following economy:

Time: Discrete; infinite horizon

Demography: Continuum of mass 1 of (representative) households that live for ever.

Preferences: The instantaneous household utility function over, home good consumption, c^h, and market good consumption, c^m, is $u(c^h, c^m)$. $u(.,.)$ is twice differentiable, strictly increasing in both arguments and is strictly concave. We also assume $u_{12}(c^h, c^m) > 0$ and

$$\begin{aligned} \lim_{c^h \to 0} u_1(c^h, c^m) &= \infty \text{ for all } c^m > 0, \\ \lim_{c^m \to 0} u_2(c^h, c^m) &= \infty \text{ for all } c^h > 0, \end{aligned}$$

where u_i represents the derivative of u with respect to the ith argument. These simply mean that household always seek to keep both c^h and c^m positive (no corner solutions). The discount factor is $\beta \in (0, 1)$.

Technology: Each household owns a technology, $f(k)$, which they cannot sell. The technology utilizes capital, k, obtained from foregone consumption to create home goods. Capital completely depreciates in one period (i.e. $\delta = 1$). Capital acquired this period is used for production next period.

Endowments: Households' initial capital stock is k_0. Each household has an initial stock, H_0 of cash.

Institutions: Every period there are markets in which cash is traded for home and market goods. Home goods produced in period t can be consumed (entering the utility function as c_t^h), or they can traded for money. To obtain market goods, c_{t+1}^m, or capital, k_{t+1}, the household has to acquire money this period and trade it for market goods in the next period (cash-in-advance). Market goods are therefore simply someone else's home goods. Market goods acquired in period t can either be consumed (entering the utility function as c_t^m) or they can be converted one-for-one into units of capital, k_t.

There is a government that has the power to levy taxes or make transfers of cash, τ_t, by helicopter drop. They grow the money supply at the rate σ so that the per household stock of cash at time t is given by

$$H_t = H_0 (1 + \sigma)^t .$$

(a) Write down the problem faced by a representative household in period t in recursive form.

(b) Derive the necessary conditions for a solution to the household's problem and the envelope conditions.

(c) Write down the market clearing conditions, the government budget constraint, and define a competitive equilibrium

(d) Show that a (strictly) binding cash-in-advance constraint means that households cannot consume efficient amounts of the two goods. Explain your answer.