Macroeconomics

Partial Answer Key

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements*, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer.

1. Since investment equals savings, an increase in the savings rate should help bring the economy out of recession.

2. In the recent "cash for clunkers" program, the federal government gave a $4,500 subsidy to a car buyer who traded in a gas-guzzling vehicle to buy a new, more fuel-efficient vehicle. Sales boomed. This surge in sales will jump start the auto industry, and one should expect a quick recovery of the industry.

3. Consumption smoothing implies that the life-cycle pattern of consumption should be independent of the life-cycle pattern of income.

4. Improvements in technology are often modelled as increases in total factor productivity for a constant returns to scale (CRS) production function. However, as firms with CRS technologies do not make any profits, there is no reason for any one to adopt the new technology.

5. Battling the financial crisis, the Federal Reserve has bailed out many banks, by purchasing hard-to-sell, illiquid assets. The banks have not invested the money received, but have kept it as excess reserves. As the economy recovers, to avoid inflation the Federal Reserve can prevent money supply growth: it can increase the interest it pays on bank reserves, so the banks will hold on to their reserves

6. When prices and wages are completely flexible, changes in the money supply have no real effects.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. **Optimal growth with preference for leisure.**

Consider the following economy:

- **Time:** Discrete; infinite horizon
- **Demography:** A mass 1 of infinite-lived households.
- **Preferences:** The present value of lifetime utility for a household is

$$\sum_{t=0}^{\infty t} \beta^t u(c_t, l_t)$$

where c_t is consumption in period t, l_t is leisure taken in period t and $\beta < 1$ is a common discount factor. The instantaneous utility function $u(.,.)$ is twice differentiable, strictly increasing in both arguments, and strictly concave with

$$\begin{aligned} \lim_{c \to 0} u_1(c, l) &= \infty, \text{ for all } l > 0, \\ \lim_{l \to 0} u_2(c, l) &= \infty \text{ for all } c > 0, \\ \lim_{c \to \infty} u_1(c, l) &= 0, \text{ for all } l > 0. \end{aligned}$$

(This just rules out corner solutions.)

- **Productive technology:** Each household has access to a neoclassical production technology $f(.,.)$ which uses physical capital, k and labor, h, to produce the consumption good. f is homogeneous of degree 1, twice differentiable, increasing in both arguments and strictly concave with $\lim_{h \to 0} f_1(k, h) = \infty$ for all $h > 0$ and $\lim_{l \to 0} f_2(k, h) = \infty$ for all $k > 0$. The consumption good can be converted one-for-one into physical capital. Physical capital depreciates by the fraction δ for each period it is in use.
- **Endowments:** Each household has one unit of time in each period which they can divide between providing labor and taking leisure. Households have an initial capital stock, k_0.

(a) The Social Planner's problem for a representative household is

$$\max_{\{c_t, l_t, k_{t+1}, h_t, i_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$
$$\begin{aligned} \text{subject to: } & f(k_t, h_t) = c_t + i_t, \\ & k_{t+1} = i_t + (1 - \delta) k_t, \\ & l_t = 1 - h_t. \end{aligned}$$

or

$$\max_{\{k_{t+1}, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(f(k_t, h_t) - k_{t+1} + (1 - \delta) k_t, 1 - h_t).$$

The first-order conditions are

$$\begin{aligned} k_t &: \beta^t u_1(c_t, l_t)[f_1(k_t, h_t) + (1-\delta)] - \beta^{t-1} u_1(c_{t-1}, l_{t-1}) = 0, \\ h_t &: \beta^t u_1(c_t, l_t) f_2(k_t, h_t) - \beta^t u_2(c_t, l_t) = 0. \end{aligned}$$

(b) We can characterize the Planner's solution by simplifying the first-order conditions:

$$\begin{aligned} \frac{u_1(c_t, 1-h_t)}{u_1(c_{t-1}, 1-h_{t-1})} &= \frac{1}{\beta[f_1(k_t, h_t) + (1-\delta)]}, \\ u_1(c_t, 1-h_t) f_2(k_t, h_t) &= u_2(c_t, 1-h_t), \\ c_t &= f(k_t, h_t) - k_{t+1} + (1-\delta)k_t. \end{aligned}$$

- **Now consider the decentralized economy:** There are competitive markets for labor and capital services, for which the prices in terms of the consumption good are w_t and r_t, respectively.

(c) The representative household, renting out capital and selling labor services to firms, solves the following problem

$$\begin{aligned} \max_{\{c_t, l_t, k_{t+1}, h_t\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \\ \text{subject to: } & r_t k_t + w_t h_t = c_t + i_t, \\ & k_{t+1} = i_t + (1-\delta)k_t, \\ & l_t = 1 - h_t. \end{aligned}$$

or

$$\max_{\{k_{t+1}, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(r_t k_t + w_t h_t - k_{t+1} + (1-\delta)k_t, 1-h_t)$$

Taking first-order conditions and simplifying, the solution to the household's problem can be characterized as

$$\begin{aligned} \frac{u_1(c_t, 1-h_t)}{u_1(c_{t-1}, 1-h_{t-1})} &= \frac{1}{\beta[r_t + (1-\delta)]} \\ u_1(c_t, 1-h_t) w_t &= u_2(c_t, 1-h_t) \\ c_t &= r_t k_t + w_t h_t - k_{t+1} + (1-\delta)k_t \end{aligned}$$

(d) The price-taking firm solves

$$\max_{k_t^f, h_t^f} f(k_t^f, h_t^f) - r_t k_t^f - w_t h_t^f.$$

The resulting first-order conditions are

$$\begin{aligned} f_1(k_t^f, h_t^f) &= r_t, \\ f_2(k_t^f, h_t^f) &= w_t. \end{aligned}$$

(e) The market clearing conditions for this economy are

$$k_t^f = k_t, \; h_t^f = h_t.$$

A competitive equilibrium is an allocation $\{c_t, l_t, k_{t+1}, h_t, k_t^f, h_t^f\}_{t=0}^{\infty}$ and a sequence of prices $\{r_t, w_t\}_{t=0}^{\infty}$ such that: given prices the allocation solves the households' and firms' problems; and markets clear

(f) We can show that an equilibrium has the same characterization as the solution to the planner's problem in two steps. First, use the market clearing conditions to replace k_t^f and h_t^f in the firm's first-order conditions (in part (d)). Second, use the modified first-order conditions to eliminate w_t and r_t from the household's first-order conditions (in part (c)). The expressions that result will be identical to those in part (b) .

8. The production function for this economy is given by

$$Y_t = L_t^{1-\alpha}, \quad 0 < \alpha < 1. \tag{PRF}$$

The preferences of the representative household are

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln \left(C_t + \gamma_t G \right) - \chi L_t \right] \right),$$

$$0 < \beta < 1, \quad 0 \leq \gamma \leq 1, \quad \chi > 0,$$

with the preference shifter γ_t following an AR(1) process around the log of its steady state value:

$$\widehat{\gamma}_t \equiv \ln \left(\gamma_t / \gamma_{ss} \right) = \phi \widehat{\gamma}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \tag{TS}$$

where $\{\varepsilon_t\}$ is an exogenous i.i.d. process, and $0 < \gamma_{ss} < 1$.

The capital accumulation equation is

$$K_{t+1} = (1 + r) K_t + L_t^{1-\alpha} - C_t - G. \tag{CA}$$

(a) In recursive form, the social planner's problem is

$$V(K_t, \gamma_t) = \max_{\{C_t, L_t\}} \ln (C_t + \gamma_t G) - \chi L_t + \beta E_t \left(V \left((1 + r) K_t + L_t^{1-\alpha} - C_t - G, \gamma_{t+1} \right) \right).$$

The first order conditions are

$$\frac{1}{C_t + \gamma_t G} = \beta E_t \left(\frac{\partial V(K_{t+1}, \gamma_{t+1})}{\partial K_{t+1}} \right), \tag{FOC1}$$

$$\chi = \beta E_t \left(\frac{\partial V(K_{t+1}, \gamma_{t+1})}{\partial K_{t+1}} \right) (1 - \alpha) L_t^{-\alpha}. \tag{FOC2}$$

Using Benveniste and Scheinkman's results, we find that

$$\frac{\partial V(K_t, \gamma_t)}{\partial K_t} = \beta E_t \left(\frac{\partial V(K_{t+1}, \gamma_{t+1})}{\partial K_{t+1}} \right) (1 + r).$$

Inserting equation (FOC1), this reduces to

$$\frac{\partial V(K_t, \gamma_t)}{\partial K_t} = \frac{1}{C_t + \gamma_t G}(1+r),$$

and (FOC1) becomes

$$\frac{1}{C_t + \gamma_t G} = \beta(1+r) E_t\left(\frac{1}{C_{t+1} + \gamma_{t+1} G}\right) = E_t\left(\left(\frac{1}{C_{t+1} + \gamma_{t+1} G}\right)\right). \qquad \text{(EE)}$$

Combining equations (FOC1) and (FOC2) yields

$$\chi L_t^{\alpha} = (1-\alpha)\frac{1}{C_t + \gamma_t G}, \qquad \text{(LL)}$$

The capital accumulation equation (CA) was derived above when formulating the social planner's problem.

(b) Logging both sides of equation (LL1′) yields

$$\begin{aligned} \ln(\chi) + \alpha \ln L_t &= \ln(1-\alpha) - \ln(C_t + \gamma_t G) \\ &= \ln(1-\alpha) - \ln[\exp(\ln C_t) + \exp(\ln \gamma_t) G]. \end{aligned}$$

Implicitly differentiating this expression yields

$$\alpha d \ln L_t = -\frac{1}{C_t + \gamma_t G}[C_t d \ln C_t + G\gamma_t d \ln \gamma_t].$$

Let lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values. The preceding equation simplifies to

$$\alpha \widehat{\ell}_t \approx -\frac{1}{C_{SS} + \gamma_{SS} G}[C_{SS}\widehat{c}_t + G\gamma_{SS}\widehat{\gamma}_t],$$

so that

$$\widehat{\ell}_t = -\frac{1}{\alpha}[\theta \widehat{c}_t + \lambda \widehat{\gamma}_t], \qquad \text{(LL}'\text{)}$$

$$\theta \equiv \left(\frac{C_{SS}}{C_{SS} + \gamma_{SS} G}\right) > 0, \quad \lambda \equiv \left(\frac{\gamma_{SS} G}{C_{SS} + \gamma_{SS} G}\right) > 0.$$

Proceeding similarly, it follows from equation (PRF′) that

$$\exp(\ln Y_t) = \exp((1-\alpha)\ln L_t),$$

so that

$$\widehat{y}_t = (1-\alpha)\widehat{\ell}_t = -\frac{1-\alpha}{\alpha}[\theta \widehat{c}_t + \lambda \widehat{\gamma}_t]. \qquad \text{(PRF}'\text{)}$$

With $\gamma_{SS} \in (0,1)$ and $G, C_{SS} > 0$, we see that $\lambda, \mu \in (0,1)$. A positive value of λ means that, controlling for private consumption (so there are no wealth effects), an increase in the consumption value of government spending reduces labor (and thus output). As government spending becomes a better substitute for private spending, total composite consumption increases. This in turn reduces the marginal utility of private (and government) consumption, lowering the consumer's marginal benefit to working and reducing hours.

(c) Implicitly differentiate equation (CA):

$$\begin{aligned}
\exp(\ln K_{t+1}) &= (1+r)\exp(\ln K_t) + \exp(\ln Y_t) - \exp(\ln C_t) - \exp(\ln G), \\
\exp(\ln K_{t+1})\, d\ln K_{t+1} &= (1+r)\exp(\ln K_t)\, d\ln K_t + \exp(\ln Y_t)\, d\ln Y_t \\
&\quad - \exp(\ln C_t)\, d\ln C_t - \exp(\ln G).
\end{aligned}$$

so that

$$K_{ss}\widehat{k}_{t+1} \approx (1+r) K_{ss}\widehat{k}_t + Y_{ss}\widehat{y}_t - C_{ss}\widehat{c}_t$$

With $C_{ss}/K_{ss} = \psi$, $G_{ss}/K_{ss} = \pi$ and $Y_{ss}/K_{ss} = \psi + \pi - r$, this simplifies to

$$\begin{aligned}
\widehat{k}_{t+1} &\approx (1+r)\widehat{k}_t + \frac{Y_{ss}}{K_{ss}}\widehat{y}_t - \frac{C_{ss}}{K_{ss}}\widehat{c}_t \\
&= (1+r)\widehat{k}_t + (\psi + \pi - r)\widehat{y}_t - \psi\widehat{c}_t.
\end{aligned}$$

Then insert (PRF′) to get

$$\widehat{k}_{t+1} \approx (1+r)\widehat{k}_t - (\psi + \pi - r)\frac{1-\alpha}{\alpha}\left[\theta\widehat{c}_t + \lambda\widehat{\gamma}_t\right] - \psi\widehat{c}_t,$$

so that the log-linear approximation for the capital accumulation equation is

$$\widehat{k}_{t+1} = (1+r)\widehat{k}_t - \omega_1\widehat{\gamma}_t - \omega_2\widehat{c}_t, \qquad \text{(CA}'\text{)}$$

$$\omega_1 = (\psi + \pi - r)\frac{1-\alpha}{\alpha}\lambda, \quad \omega_2 = \psi + (\psi + \pi - r)\frac{1-\alpha}{\alpha}\theta.$$

(d) One can log-linearize the Euler equation and solve the resulting system to express consumption as a function of capital and government spending:

$$\begin{aligned}
\widehat{c}_t &= \eta\widehat{k}_t - \mu\widehat{\gamma}_t, \qquad \text{(CF)} \\
\eta &> 0, \quad 0 < \mu < \frac{\lambda}{\theta}.
\end{aligned}$$

1. Combining equations (PRF′) and (CF) shows that

$$\widehat{y}_t = -\frac{1-\alpha}{\alpha}\left[\theta\eta\widehat{k}_t + (\lambda - \theta\mu)\widehat{\gamma}_t\right],$$

   Continuing, the log of average labor productivity is given by

$$\begin{aligned}
\widehat{apl}_t &= \widehat{y}_t - \widehat{\ell}_t \\
&= \theta\eta\widehat{k}_t + (\lambda - \theta\mu)\widehat{\gamma}_t,
\end{aligned}$$

2. Given $\mu < \lambda/\theta$, we have $\lambda - \theta\mu > 0$, so that $\widehat{y}_t$ is decreasing in $\widehat{\gamma}_t$, while $\widehat{apl}_t$ is increasing. If there is an unexpected increase in the utility received from government goods ($\widehat{\gamma}$ *rises*), output will initially fall, while the average product of labor will rise. Extrapolating from the initial response, we thus expect labor productivity to be counter-cyclical in this model. Intuitively, an increase in $\widehat{\gamma}$ has a wealth effect that reduces labor and thus output. With decreasing returns to labor, however, a decrease in labor causes average productivity, Y/L, to rise.

(e) Attached are the background derivations—**not** needed for the answers above—behind the shortcuts provided in the exam.

Using calculations are identical to those shown in the answer to question 7 of the June, 2007 comprehensive exam, we can show that

$$\begin{aligned} \eta &= \frac{r}{\omega_2}, \\ \mu &= \frac{1}{\theta\,(1-\phi+r)}\left[\lambda\,(1-\phi)+\theta r\frac{\omega_1}{\omega_2}\right]. \end{aligned}$$

Now it follows from the answer to part (d) above that

$$\begin{aligned} \frac{\omega_1}{\omega_2} &= \left[(\psi+\pi-r)\,\frac{1-\alpha}{\alpha}\lambda\right] / \left[\psi+(\psi+\pi-r)\,\frac{1-\alpha}{\alpha}\theta\right] \\ &< \left[(\psi+\pi-r)\,\frac{1-\alpha}{\alpha}\lambda\right] / \left[(\psi+\pi-r)\,\frac{1-\alpha}{\alpha}\theta\right] = \frac{\lambda}{\theta}. \end{aligned}$$

Inserting this into the expression for μ shows that

$$\mu < \frac{1}{\theta\,(1-\phi+r)}\left[\lambda\,(1-\phi)+\theta r\frac{\lambda}{\theta}\right] = \frac{\lambda}{\theta}.$$

9. In this problem we add a new asset to an otherwise standard Lucas tree model. The preferences of the representative consumer are

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\ln(c_t)\right), \quad 0<\beta<1.$$

Output is produced by an infinite-lived tree. The tree's profits, d_t, take on the values $\{d, \gamma d\}$, where $\gamma > 1$. Profits follow an i.i.d. process with

$$\begin{aligned} f\,(d) &= \Pr\left(d_{t+1}=d\right|\left.d_t=d\right)=\Pr\left(d_{t+1}=d\right|\left.d_t=\gamma d\right) \\ &= \pi \\ f\,(\gamma d) &= \Pr\left(d_{t+1}=\gamma d\right|\left.d_t=d\right)=\Pr\left(d_{t+1}=\gamma d\right|\left.d_t=\gamma d\right) \\ &= 1-\pi. \end{aligned}$$

In addition to the "ideal stock" found in most asset pricing models, the consumer can purchase "common stock", which allows her to act as a residual claimant. Each period a unit of common stock provides a dividend, g_t, as follows:

$$g_t = \begin{cases} (\gamma-1)d, & d_t=\gamma d \\ 0, & d_t = d \end{cases}.$$

As a matter of notation, let $q_t = q_t\,(d_t)$ be the price at time t to all future dividends from a unit of common stock, and let e_t denote the number of units of common stock held by the representative consumer.

(a) Writing the consumer's problem as a Lagrangean, we get

$$V(x_t, d_t) =$$
$$\min_{\lambda_t \geq 0} \max_{c_t \geq 0,\ b_{t+1},\ s_{t+1},\ e_{t+1}} \ln(c_t) + \lambda_t \left(x_t - c_t - p_t s_{t+1} - q_t e_{t+1}\right)$$
$$+ \beta \sum_{d_{t+1} \in \{d, \gamma d\}} f\left(d_{t+1}\right) \times V\left(\left(\begin{array}{c} b_{t+1} + \left[p_{t+1}\left(d_{t+1}\right) + d_{t+1}\right] s_{t+1} \\ + \ \left[q_{t+1}\left(d_{t+1}\right) + g_{t+1}\left(d_{t+1}\right)\right] e_{t+1} \end{array}\right), \quad d_{t+1}\right).$$

The FOC for an interior solution are:

$$\begin{aligned} \frac{1}{c_t} &= \lambda_t, \\ \lambda_t p_t &= \beta \sum_{d_{t+1} \in \{d, \gamma d\}} f\left(d_{t+1}\right) \times \frac{\partial V\left[t+1\right]}{\partial x_{t+1}} \left[p_{t+1}\left(d_{t+1}\right) + d_{t+1}\right], \\ \lambda_t q_t &= \beta \sum_{d_{t+1} \in \{d, \gamma d\}} f\left(d_{t+1}\right) \times \frac{\partial V\left[t+1\right]}{\partial x_{t+1}} \left[q_{t+1}\left(d_{t+1}\right) + g_{t+1}\left(d_{t+1}\right)\right]. \end{aligned}$$

Since (following Benveniste-Scheinkman),

$$\frac{\partial V\left[t\right]}{\partial x_t} = \lambda_t,$$

the Euler equations are

$$p_t = \beta E_t \left(\frac{c_t}{c_{t+1}} \left[p_{t+1}\left(d_{t+1}\right) + d_{t+1}\right]\right), \tag{EE1}$$

$$q_t = \beta E_t \left(\frac{c_t}{c_{t+1}} \left[q_{t+1}\left(d_{t+1}\right) + g_{t+1}\left(d_{t+1}\right)\right]\right). \tag{EE2}$$

(b) In equilibrium, $c_t = d_t$. Since $0 < \beta < 1$, it makes sense to solve equation (EE1) forward:

$$E_t \left(\left(1 - \beta L^{-1}\right) p_t \frac{1}{d_t}\right) = \beta E_t \left(\frac{d_{t+1}}{d_{t+1}}\right),$$

so that

$$p_t \frac{1}{d_t} = \frac{1}{1 - \beta L^{-1}} \beta + b_t = \frac{\beta}{1 - \beta} + b_t,$$

with the "bubble term" b_t obeying

$$E_t\left(b_{t+1}\right) = \beta^{-1} b_t.$$

We now require that

$$\lim_{J \to \infty} E_t\left(\beta^J p_{t+J}\right) = 0, \quad \forall t. \tag{TVC}$$

(TVC) will be satisfied only if $b_t = 0$ and the price of an "ideal" stock is

$$p_t = p\left(d_t\right) = \frac{\beta}{1 - \beta} d_t \equiv \overline{p} d_t. \tag{EQP}$$

(c) To find the price of an ideal stock, we solve equation (EE2) forward:

$$\begin{aligned} q_t \frac{1}{d_t} &= \frac{1}{1-\beta L^{-1}} \beta E_t \left(\frac{g_{t+1}}{d_{t+1}} \right) + b_t \\ &= \beta E_t \left(\sum\nolimits_{j=0}^{\infty} \beta^j \frac{g_{t+1+j}}{d_{t+1+j}} \right), \end{aligned}$$

so that

$$q_t = d_t E_t \left(\sum\nolimits_{j=1}^{\infty} \beta^j \frac{g_{t+j}}{d_{t+j}} \right).$$

Because profits are i.i.d., the expectation is time-invariant, and we have

$$\begin{aligned} q_t &= d_t E_t \left(\sum\nolimits_{j=1}^{\infty} \beta^j \left[\pi \cdot \frac{0}{d} + (1-\pi) \frac{(\gamma-1)\, d}{\gamma d} \right] \right) \\ &= d_t \frac{\beta}{1-\beta} (1-\pi) \left(\frac{\gamma-1}{\gamma} \right) d_t = (1-\pi) \left(\frac{\gamma-1}{\gamma} \right) \overline{p} d_t \\ &\equiv \overline{q} d_t. \end{aligned} \tag{EQQ}$$

(d) When current dividends are low ($d_t = d$), it follows from equation (EQP) that the expected return on ideal stock is

$$\begin{aligned} E\left(R_t^s \middle| d_t = d \right) &= E \left(\frac{p_{t+1} + d_{t+1}}{p_t} \middle| d_t = d \right) \\ &= \frac{1}{\overline{p} d} \left(\pi \left[\overline{p} d + d \right] + (1-\pi) \left[\overline{p} \gamma d + \gamma d \right] \right) \\ &= \frac{\overline{p}+1}{\overline{p}} \left(\pi + (1-\pi) \gamma \right) \\ &= \pi + (1-\pi)\gamma + \frac{1}{\overline{p}} \left(\pi + (1-\pi)\gamma \right). \end{aligned}$$

The expected return on common stock is derived similarly:

$$\begin{aligned} E\left(R_t^e \middle| d_t = d \right) &= E \left(\frac{q_{t+1} + g_{t+1}}{q_t} \middle| d_t = d \right) \\ &= \frac{1}{\overline{q} d} \left(\pi \overline{q} d + (1-\pi) \left[\overline{q} \gamma d + \gamma d \right] \right) \\ &= \pi + (1-\pi)\gamma + \frac{1-\pi}{\overline{q}\gamma} \end{aligned}$$

(e) To compare the two returns, note that

$$\begin{aligned} E\left(R_t^e \middle| d_t = d \right) - E\left(R_t^s \middle| d_t = d \right) &= \frac{1}{\overline{p}} \left(\pi + (1-\pi)\gamma \right) - \frac{1-\pi}{\overline{q}\gamma} \\ &= \frac{1}{\overline{p}} \left(\pi + (1-\pi)\gamma \right) - \frac{1}{\overline{p}(\gamma-1)}. \end{aligned}$$

As $\gamma \to \infty$, this term becomes positive, and as $\gamma \to 1$ from above, this term becomes negative. The price of ideal stock and its periodic payment are both proportional to current output. In contrast, the price of common stock is proportional to current output, but its periodic dividend is not. As a result, the covariance between realized returns on common stock and output/marginal utility of consumption is different from the covariance between output and the realized returns on ideal stock. This leads investors to demand different expected returns.

10. **Search with part and full-time jobs.**

- **Time:** Discrete, infinite horizon.
- **Demography:** Single worker who lives for ever.
- **Preferences:** The worker is risk-neutral (i.e. $u(x) = x$) and discounts the future at the rate r.
- **Endowments:**
  - While unemployed the worker gets a flow utility from leisure of $b > 0$.
  - Regardless of her employment status, each period with probability α_p she gets an offer of a part-time job that pays w_p. Or, with probability α_f she gets a full-time job offer with wage w_f. Assume that $w_f > w_p$ and that $w_f > b$. With probability $1 - \alpha_p - \alpha_f$, she gets no offer.
  - In addition to job offers, an employed worker can lose her job. The probability that this happens is λ. (Assume that $\alpha_p + \alpha_f + \lambda < 1$.)

(a) Let V_u, V_p, and V_f denote the values to being unemployed, part-time employed and full-time employed, respectively. in terms of each other. Assuming that $V_f > V_p$ (so that full-time employees ignore offers of part-time jobs), the asset value equations are

$$\begin{aligned} rV_u &= b + \alpha_p(V_p - V_u) + \alpha_f(V_f - V_u), \\ rV_p &= w_p + \alpha_f(V_f - V_p) + \lambda(V_u - V_p), \\ rV_f &= w_f + \lambda(V_u - V_f). \end{aligned}$$

(b) To find the value of w_p at which workers are just indifferent between part-time employment and unemployment, let $V_u = V_p = V^*$. Then the first 2 equations become

$$\begin{aligned} rV^* &= b + \alpha_f(V_f - V^*) \\ rV^* &= w_p + \alpha_f(V_f - V^*) \end{aligned}$$

which implies $w_p = b$.

- Now suppose instead that while employed part-time, workers get full-time job offers at the rate $\alpha_e \neq \alpha_f$. (Assume that $\alpha_p + \alpha_e + \lambda < 1$.)

(c) The revised asset value equations are

$$
\begin{aligned}
rV_u &= b + \alpha_p(V_p - V_u) + \alpha_f(V_f - V_u), \\
rV_p &= w_p + \alpha_e(V_f - V_p) + \lambda(V_u - V_p), \\
rV_f &= w_f + \lambda(V_u - V_f).
\end{aligned}
$$

Nothing else changes.

(d) To recalculate the value of w_p at which workers are just indifferent between part-time employment and unemployment, we again set $V_u = V_p = V^*$. Then the first 2 equations of part (c) become

$$
\begin{aligned}
rV^* &= b + \alpha_f(V_f - V^*), \\
rV^* &= w_p + \alpha_e(V_f - V^*).
\end{aligned}
$$

If $\alpha_e > \alpha_f$, we need $w_p < b$ for indifference between unemployment and part-time work. If $\alpha_e > \alpha_f$ we need $w_p > b$.

(e) Our findings in part (d) can be explained as follows. When the offer rate for the higher-paid job is higher for those in part-time work, there is an option value to part-time jobs because they give access to more full-time offers. The worker's reservation wage will be below the value of leisure. When the offer rate for the higher-paid job is higher for the unemployed, there is an option value to remaining in unemployment for which part-time employers must compensate.