Macroeconomics

Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements*, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.

1. Since investment equals savings, an increase in the savings rate should help bring the economy out of recession.

2. In the recent "cash for clunkers" program, the federal government gave a $4,500 subsidy to a car buyer who traded in a gas-guzzling vehicle to buy a new, more fuel-efficient vehicle. Sales boomed. This surge in sales will jump start the auto industry, and one should expect a quick recovery of the industry.

3. Consumption smoothing implies that the life-cycle pattern of consumption should be independent of the life-cycle pattern of income.

4. Improvements in technology are often modelled as increases in total factor productivity for a constant returns to scale (CRS) production function. However, as firms with CRS technologies do not make any profits, there is no reason for any one to adopt the new technology.

5. Battling the financial crisis, the Federal Reserve has bailed out many banks, by purchasing hard-to-sell, illiquid assets. The banks have not invested the money received, but have kept it as excess reserves. As the economy recovers, to avoid inflation the Federal Reserve can prevent money supply growth: it can increase the interest it pays on bank reserves, so the banks will hold on to their reserves

6. When prices and wages are completely flexible, changes in the money supply have no real effects.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. **Optimal growth with preference for leisure.**

Consider the following economy:

- **Time:** Discrete; infinite horizon
- **Demography:** A mass 1 of infinite-lived households.
- **Preferences:** The present value of lifetime utility for a household is

$$\sum_{t=0}^{\infty t} \beta^t u(c_t, l_t)$$

where c_t is consumption in period t, l_t is leisure taken in period t and $\beta < 1$ is a common discount factor. The instantaneous utility function $u(.,.)$ is twice differentiable, strictly increasing in both arguments, and strictly concave with

$$\lim_{c \to 0} u_1(c, l) = \infty, \text{ for all } l > 0,$$
$$\lim_{l \to 0} u_2(c, l) = \infty \text{ for all } c > 0,$$
$$\lim_{c \to \infty} u_1(c, l) = 0, \text{ for all } l > 0.$$

(This just rules out corner solutions.)

- **Productive technology:** Each household has access to a neoclassical production technology $f(.,.)$ which uses physical capital, k and labor, h, to produce the consumption good. f is homogeneous of degree 1, twice differentiable, increasing in both arguments and strictly concave with $\lim_{h \to 0} f_1(k, h) = \infty$ for all $h > 0$ and $\lim_{l \to 0} f_2(k, h) = \infty$ for all $k > 0$. The consumption good can be converted one-for-one into physical capital. Physical capital depreciates by the fraction δ for each period it is in use.
- **Endowments:** Each household has one unit of time in each period which they can divide between providing labor and taking leisure. Households have an initial capital stock, k_0.

(a) Write down the problem faced by the Social Planner for a representative household.

(b) Characterize a solution by simplifying the Planner's first-order conditions.

- **Now consider the decentralized economy:** There are competitive markets for labor and capital services, for which the prices in terms of the consumption good are w_t and r_t, respectively.

(c) Write down and characterize the solution to the problem faced by a representative household which rents its capital and sells its labor services to firms.

(d) Write down and characterize the solution to the problem faced by a firm.

(e) Write down the market clearing conditions and define an equilibrium.

(f) Show that an equilibrium shares the same characterization as the solution to the planner's problem.

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8. (Extended from Christiano and Eichenbaum, 1992.) Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,$$

$$Y_t = L_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{PRF}$$

where: Π_t is profit; L_t is labor; W_t is the real wage; and Y_t is output. The population and number of firms are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption, C_t, government spending, $G_t = G$, and labor are given by

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln \left(C_t + \gamma_t G \right) - \chi L_t \right] \right),$$

$$0 < \beta < 1, \quad \chi > 0.$$

Households receive labor income and profits from firms. They pay lump-sum taxes, $H_t = H$, to the government. Households earn a gross return of $(1+r) K_t$ on their assets, K_t, with $\beta (1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1}, are chosen in period t. Note that in this economy, capital is used only as a storage device. Households also face the usual non-negativity and No-Ponzi-Game conditions.

The government's flow budget constraint is given by

$$H_t = G_t \Leftrightarrow H = G.$$

Although government spending is constant, the utility consumers receive from government spending will vary. In particular, the preference parameter γ_t, which gives the rate at which government spending substitutes for private consumption, follows an AR(1) process around the log of its steady state value:

$$\widehat{\gamma}_t \equiv \ln \left(\gamma_t / \gamma_{ss} \right) = \phi \widehat{\gamma}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \tag{TS}$$

where $\{\varepsilon_t\}$ is an exogenous i.i.d. process, and $0 < \gamma_{ss} < 1$. (You can also assume that $C_t + \gamma_t G > 0, \ \forall t$.)

(a) Write down the social planner's problem for this economy as a dynamic programming problem. (The planner takes $\gamma_t G$ as given.) Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation.

(b) Let lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$\widehat{\ell}_t = -\frac{1}{\alpha} \left[\theta \widehat{c}_t + \lambda \widehat{\gamma}_t \right],$$

$$\widehat{y}_t = -\frac{1-\alpha}{\alpha} \left[\theta \widehat{c}_t + \lambda \widehat{\gamma}_t \right].$$

Show that $\lambda, \theta \in (0,1)$. Why is λ positive? Provide an intuitive justification.

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(c) Suppose that $C_{ss}/K_{ss} = \psi$, while $G_{ss}/K_{ss} = \pi$, with $\psi + \pi > r$, which implies that Y_{ss}/K_{ss} is $\psi + \pi - r$. Using this result, log-linearize the capital accumulation equation to show that

$$\widehat{k}_{t+1} = (1 + r)\,\widehat{k}_t - \omega_1\widehat{\gamma}_t - \omega_2\widehat{c}_t, \quad \omega_1, \omega_2 > 0.$$

(d) One can log-linearize the Euler equation and solve the resulting system (which also includes (TS)) to express consumption as a function of capital and the preference shock:

$$
\begin{aligned}
\widehat{c}_t &= \eta\widehat{k}_t - \mu\widehat{\gamma}_t, &&\text{(CF)}\\
\eta &> 0, \quad 0 < \mu < \frac{\lambda}{\theta}.
\end{aligned}
$$

(Take all of this as given.)

1. Consider the log of average labor productivity,

$$\widehat{apl}_t = \widehat{y}_t - \widehat{\ell}_t,$$

Express $\widehat{y}_t$ and $\widehat{apl}_t$ as functions of capital and the preference shock $\widehat{\gamma}_t$.

2. Suppose there is an unexpected increase in the utility received from government goods ($\widehat{\gamma}$ *rises*). In what direction will output, average labor productivity and measured labor productivity initially respond? Extrapolating from the initial response, would you expect labor productivity to be pro- or counter-cyclical in this model? Briefly explain.

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9. Consider the following variant of the Lucas tree model. The preferences of the representative consumer are

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \ln(c_t) \right), \quad 0 < \beta < 1,$$

where c_t denotes consumption, and $E_t(\cdot)$ denotes expectations conditional on time-t information.

Output is produced by an infinite-lived tree: each period, the tree produces d_t units of non-storable output. Profits, d_t, take on the values $\{d, \gamma d\}$, where $\gamma > 1$. Profits follow an i.i.d. process with

$$
\begin{aligned}
f(d) &= \Pr(d_{t+1} = d | d_t = d) = \Pr(d_{t+1} = d | d_t = \gamma d) \\
&= \pi \\
f(\gamma d) &= \Pr(d_{t+1} = \gamma d | d_t = d) = \Pr(d_{t+1} = \gamma d | d_t = \gamma d) \\
&= 1 - \pi.
\end{aligned}
$$

The economy starts off with each household owning one tree apiece.

Let $p_t = p_t(d_t)$ be the price at time t of all future profits from a tree, which we will call a unit of "ideal stock". In addition, consumers have access to "common stock", which allows them to act as residual claimants. Each period a unit of common stock provides a dividend, g_t, as follows:

$$g_t = \begin{cases} (\gamma - 1)d, & \text{if } d_t = \gamma d \\ 0, & \text{if } d_t = d \end{cases}.$$

Let $q_t = q_t(d_t)$ be the price at time t to all future dividends from a unit of common stock, and let e_t denote the number of units of common stock held by the representative consumer. Finally, let x_t denote the consumer's financial resources, which she allocates between ideal stock, common stock, and consumption.

(a) Write down the consumer's problem in recursive form and find the first order conditions.

(b) Find the equilibrium price for ideal stock, $p(d_t)$, and show that it can be written as $\overline{p}d_t$.

(c) Find the equilibrium price for common stock, $q(d_t)$, and show that it can be written as $\overline{q}d_t = (1 - \pi)\left(\dfrac{\gamma - 1}{\gamma}\right)\overline{p}d_t$.

(d) Let R_t^s denote the realized rate of return on ideal stock, and let R_t^e denote the realized rate of return on common stock. Find the expected rates of returns on the two types of stock when current income is low, that is, find $E(R_t^s | d_t = d)$ and $E(R_t^e | d_t = d)$.

(e) Compare the expected returns on the two types of stocks. Provide an intuitive explanation of your findings.

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10. **Search with part and full-time jobs.**

- **Time:** Discrete, infinite horizon.
- **Demography:** Single worker who lives for ever.
- **Preferences:** The worker is risk-neutral (i.e. $u(x) = x$) and discounts the future at the rate r.
- **Endowments:**
  - While unemployed the worker gets a flow utility from leisure of $b > 0$.
  - Regardless of her employment status, each period with probability α_p she gets an offer of a part-time job that pays w_p. Or, with probability α_f she gets a full-time job offer with wage w_f. Assume that $w_f > w_p$ and that $w_f > b$. With probability $1 - \alpha_p - \alpha_f$, she gets no offer.
  - In addition to job offers, an employed worker can lose her job. The probability that this happens is λ. (Assume that $\alpha_p + \alpha_f + \lambda < 1$.)

(a) Write out the asset value equations for each of V_u, V_p, and V_f (the values to being unemployed, part-time employed and full-time employed, respectively), in terms of each other. (Note: you can assume that $V_f > V_p$, so that full-time employees ignore offers of part-time jobs.)

(b) Solve for the value of w_p at which workers are just indifferent between part-time employment and unemployment. (You do not need to solve all the value functions.)

- Now suppose instead that while employed part-time, workers get full-time job offers at the rate $\alpha_e \neq \alpha_f$. (Assume that $\alpha_p + \alpha_e + \lambda < 1$.)

(c) Rewrite the value functions to reflect this change.

(d) How do the relative sizes of α_e and α_f affect the value of w_p at which workers are just indifferent between part-time employment and unemployment?

(e) Explain your answer to part (d).