

Macroeconomics

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.*

1. Money-in-the-utility-function is a superior modelling technique to cash-in-advance because the model predictions are basically the same but the former approach is more tractable (i.e. easier to analyze).
2. Ricardian equivalence cannot hold in any model that incorporates birth and death (e.g. overlapping generations).
3. Fiscal stimulus increases production.
4. Assuming that government spending is used to produce public goods which increase utility, pro-cyclical government spending is welfare-improving compared to a-cyclical or counter-cyclical government spending.
5. Although the Federal Reserve has dramatically increased the money supply, unemployment in the U.S. remains high. This shows that money is neutral (and superneutral).
6. In a famous article, Robert Shiller found that the prices of stocks are more volatile than the expected present value of the stocks' dividend streams. This shows that stock prices experience bubbles.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. **One-sided search with technological aging**

Time: Discrete, infinite horizon.

Demography: Single worker who lives for ever.

Preferences: The worker is risk-neutral (i.e. $u(x) = x$) and discounts the future at the rate r .

Endowments:

While unemployed she gets a flow utility from leisure of $b > 0$. With probability α she also gets a job offer. All offers are made at the same wage $w > b$ (no need for a distribution function).

Hiring only occurs at “young” firms. Working for a young firm means that the probability the worker gets laid off in any period is λ_y . In any period working at a young firm, with probability γ the worker’s employer becomes an “old” firm. Working for an old firm means that the probability the worker gets laid off in any period is $\lambda_o > \lambda_y$. (Lay-off occurs due to job destruction so old firms cannot hire unemployed workers.) The wage is w at both young and old firms. Assume that events are mutually exclusive, so that lay-off by a young firm and the firm getting old cannot happen in the same period, and $\lambda_y + \gamma < 1$.

- (a) Write down the asset value (Bellman) equations associated with each (of the 3) states the worker can be in. Briefly explain where each equation comes from.
- (b) Solve for the **flow** value equations and show that the value to employment at a young firm always exceeds the value to employment at an old firm.
- (c) Despite the result from part (b), show that the worker will not prefer to quit to unemployment when her firm becomes old. Explain why not. (Hint: use the flow value equations to show this)
- (d) Now suppose there is a continuum (mass 1) of such workers. Draw the flow diagram associated with movements between each state. Write down a set of equations that can be used to solve for the steady-state numbers (masses) of workers in each state. (You do not need to solve them.)

8. Consider the following version of the Sidrauski model. The economy is populated by a large number of identical yeoman farmers. Each farmer derives utility from consumption, leisure and real balances, according to:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) - \chi \frac{1}{1+\gamma} (\ell_t - \zeta m_t)^{1+\gamma} \right] \right), \quad \beta, \gamma, \zeta, \chi > 0, \quad \beta < 1,$$

where: c_t denotes consumption; $m_t \equiv M_t/P_t$ denotes real balances; and ℓ_t denotes time spent working. The farmer's resources evolve according to

$$M_t + P_t c_t = P_t(\tau_t + y_t) + M_{t-1}, \quad (\text{NFBC})$$

$$y_t = \ell_t^\alpha, \quad 0 < \alpha < 1, \quad (\text{PRF})$$

where: y_t denotes output; and τ_t denotes lump-sum transfers from the government. Note that **there is no capital**—and thus no investment—in this economy.

The nominal money supply M_t evolves according to

$$M_t = (1 + \theta_t)M_{t-1}, \quad (\text{MS})$$

with θ_t following an exogenous stochastic process—it is uncertainty about θ_t that requires us to work with expectations. In equilibrium, lump-sum taxes equal seigniorage:

$$\tau_t = \frac{M_t - M_{t-1}}{P_t}. \quad (\text{GBC})$$

The farmer, however, takes τ_t as given.

- Assume that $0 < m_t < \ell_t/\zeta$. Provide an intuitive justification for the assumption that money decreases hours of work in the flow utility function.
- Let $\pi_t = P_t/P_{t-1} - 1$ denote the rate of inflation. Set up the yeoman farmer's problem in **real** terms, and find the first order conditions.
- Find: the labor allocation condition; the Euler equation for money; and the resource constraint. Using the resource constraint, rewrite the labor allocation condition in terms of consumption and money.
- Let letters with carats “ $\hat{}$ ” denote deviations of logged variables around their steady state values. Using the revised labor allocation condition, show that

$$\hat{c}_t \approx \eta \hat{m}_t, \quad \eta > 0.$$

Why is η positive? Explain intuitively.

- Let “*ss*” subscripts denote steady state values.
 - Let i_{ss} denote the steady-state nominal interest rate. Using the revised labor allocation condition, and $\beta^{-1}(1 + \pi_{ss}) = 1 + i_{ss}$ (take this as given), rewrite the steady-state Euler equation to express c_{ss} as a function of i_{ss} .
 - Given your answer to part (i), what value of inflation maximizes steady-state consumption, c_{ss} ? Briefly explain.

9. Overlapping generation with patient and impatient individuals

Time: discrete, infinite horizon

Demography: A mass $N_t = N_0(1+n)^t$ of newborns enter in period t , where N_0 is the population of newborns in period 0 and n is the growth rate of the population. The share ϕ^p of each generation is patient and the share ϕ^m is impatient, with $\phi^p + \phi^m = 1$. (Variables and parameters that relate to patient individuals carry a superscript p ; those relating to impatient individuals carry a superscript m .)

Preferences: for the generations born in and after period 0;

$$U_t^j(c_{1,t}^j, c_{2,t+1}^j) = u(c_{1,t}^j) + \beta^j u(c_{2,t+1}^j), \quad j = p, m$$

where $c_{i,t}^j$ is consumption by type- j individuals in period t and stage of life i . The common instantaneous utility function $u(\cdot)$ is increasing strictly concave and twice differentiable with $\lim_{x \rightarrow 0} u'(x) = \infty$. For the initial old generation $\tilde{U}^j(c_{2,0}^j) = u(c_{2,0}^j)$. Individuals thus differ only by their lifetime discount factors: $\beta^p > \beta^m$.

Technology: $F(K, L)$ is a constant returns to scale neoclassical production technology that uses capital and labor to produce a consumption good. (Capital depreciates 100% in use.) Consumption goods can be consumed or they can be used as the capital input to production. It will be convenient to express the per (total) young population production function as $f(k_t)$, where k_t is the capital stock per young person (there is no need to distinguish who owns the capital).

Endowments: Everyone has one unit of labor services when young. When old no one can work. Every one among the initial old generation has the same k_0 units of capital which they can rent out to firms. (We can think of a single collectively-owned firm that takes wages and interest rates as given. The firm will make no profits in equilibrium.)

- (a) Write down the problems faced by each type of individual in the economy and solve for the Euler equations for each type. (There is no market for inside money.)
- (b) Show that the savings (in physical capital) of each type can be expressed (implicitly) as a function of the wage and interest rate.
- (c) Write down and solve the firm's problem to obtain the wage and interest rate as a function of the average capital holding per young person.
- (d) Write down the market clearing condition for capital (in terms of the saving functions) and define a competitive equilibrium. Write down an (implicit) expression for the steady state equilibrium capital stock, k^* .
- (e) Write down the Planner's problem (just write the Lagrangian in terms of k_t if you like) assuming the Planner treats everyone equally. Solve for the efficiency conditions.
- (f) Under what circumstances would a steady-state equilibrium in the market economy represent a Pareto optimum?
- (g) Comment on the distinction between this economy and one with a homogeneous population.

10. Consider an economy populated by a large number of identical yeoman farmers. Each farmer's preferences over consumption and leisure are:

$$E_t \left(\sum_{j=0}^{\infty} \beta^j \frac{1}{1-\gamma} c_{t+j}^{1-\gamma} (1 - \ell_{t+j}) \right), \quad 0 < \beta < 1, \quad 0 \leq \gamma < 1.$$

Let R_t^{-1} be the price of a risk-free discount bond that pays one unit of consumption at time $t + 1$, and let b_{t+1} denote the quantity of such bonds. The farmer's resources evolve according to

$$R_t^{-1} b_{t+1} + c_t = d_t \ell_t^{1-\alpha} + b_t, \quad (\text{FBC})$$

Output depends on the exogenous shifter d_t , where d_t is a non-negative random variable governed by a Markov process with the stationary transition density $f(d', d)$. Output is not storable—there is no capital.

Suppose for now that d_t is common to all farmers.

- (a) Write down the farmer's problem in recursive form and find the first order conditions.
- (b) Find equilibrium bond prices.
 - i What are equilibrium bond holdings. Why?
 - ii Show that equilibrium labor is constant, and solve for the equilibrium value ℓ_e . Intuitively, why is this the case?
 - iii Find the equilibrium expression for $R_t^{-1} = R_t^{-1}(d_t)$.
- (c) Now suppose there are two types of farms, occurring in equal proportions. The productivity process for wet-weather farms is

$$d_t^W = \delta + d_t,$$

while productivity for arid- (dry) weather farms follows

$$d_t^A = \delta - d_t.$$

d_t still follows a Markov process with a stationary transition density, but is now zero-mean and bounded by $-\delta < d_t < \delta$.

- i Assuming that the planner places equal weight on each group, set up the social planner's problem.
- ii Find the equations that characterize the optimal allocation.
- iii Is optimal labor still constant across time and farmers? Why or why not?