Macroeconomics

Section 1. (Suggested Time: 45 Minutes) *For 3 of the following 6 statements*, state whether the statement is **true**, **false**, or **uncertain**, and give a complete and convincing explanation of your answer. **Note:** Such explanations typically appeal to specific macroeconomic models.

1. Suppose a researcher finds that increases in government spending, when funded by deficits, lead to higher output and employment. Such a finding shows that countercyclical government spending is a beneficial policy.
2. An increase in the price of oil will only cause relative prices to change - not general inflation.
3. In recent months, GDP and productivity have risen, but employment has fallen. Such changes cannot be explained by a real business cycle model.
4. An increase in domestic private saving cannot be beneficial if it is simply soaked up by an increased government budget deficit.
5. Taxes on capital income are more distortionary than taxes on labor income.
6. President Obama has said that he intends to allow the "Bush tax cuts" that mainly benefit the rich to expire at the end of 2010. Allowing them to expire will worsen the current recession.

Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. **Diamond Coconut Economy with idiosyncratic preferences**

Time: Discrete, infinite horizon

Geography: A trading island and a production island.

Demography: A mass of 1 of ex ante identical individuals with infinite lives.

Preferences: The common discount rate is r, consumption of own produce yields 0 utils, consumption of someone whose good you like yields $u > 0$ utils. The share of individuals whose goods I like is ϕ. Whether an individual likes my good or not is independent of whether I like hers. (So she likes my good with probability ϕ too.)

Productive Technology: On the production island individuals come across a tree with a coconut with probability α each period. The cost of obtaining the coconut is $c \sim F$. The distribution function $F(.)$ is continuous over its support, $(0, \bar{c})$ where $\bar{c} > u$ (so some trees will get rejected).

Matching Technology: On the trading island people with coconuts meet each other with probability γ, a constant.

Navigation: Travel between islands is instantaneous.

Endowments: Everyone has a boat and starts off with one of their own coconuts.

(a) Write down the asset value (Bellman) equations for this economy. Define the terms you introduce.

The regular asset value equations are

$$\begin{aligned} V_T &= \frac{1}{1+r}\left\{\gamma\phi^2(u+V_P) + (1-\gamma\phi^2)V_T\right\} \\ V_P &= \frac{1}{1+r}\left\{\alpha \mathrm{E}\left[\max\left\{V_T - c, V_P\right\}\right] + (1-\alpha)V_P\right\} \end{aligned}$$

where V_T is the value to being on the trading island and V_P is the value to being on the production island.

The flow value equations are

$$\begin{aligned} rV_T &= \gamma\phi^2(u + V_P - V_T) \\ rV_P &= \alpha \mathrm{E}\left[\max\left\{V_T - c - V_P, 0\right\}\right]. \end{aligned}$$

(b) Define a search equilibrium.

A search equilibrium is a pattern of trade such that given everyone else conforms to that patterns no individual will wish to deviate from it.

(c) Solve for an implicit equation that specifies the reservation tree "height", c^* in terms of the parameters of the model.

Let $c^* = V_T - V_P$. Then rewrite V_P as

$$rV_P = \alpha \int_0^{c^*} \left[V_T - c - V_P\right] dF(c).$$

Now subtract rV_P from rV_T to get

$$r(V_T - V_P) = \gamma\phi^2 u - \gamma\phi^2 (V_T - V_P) - \alpha \int_0^{c^*} [V_T - c - V_P]\, dF(c)$$

or by definition of c^*,

$$(r + \gamma\phi^2)c^* = \gamma\phi^2 u - \alpha \int_0^{c^*} [c^* - c]\, dF(c). \tag{1}$$

(d) How does c^* change with respect to ϕ (i.e. obtain the sign of the comparative static)?

Define

$$\Psi(c, \phi) \equiv (r + \gamma\phi^2)c - \gamma\phi^2 u + \alpha \int_0^{c} [c - \eta]\, dF(\eta)$$

so that $\Psi(c^*, \phi) = 0$. Then

$$\frac{dc^*}{d\phi} = \left.\frac{-\frac{\partial\Psi}{\partial\phi}}{\frac{\partial\Psi}{\partial c}}\right|_{c=c^*}$$

Then

$$\left.\frac{\partial\Psi}{\partial\phi}\right|_{c=c^*} = 2\phi\gamma(c^* - u) < 0$$

(sign comes from equation (1)). And

$$\left.\frac{\partial\Psi}{\partial c}\right|_{c=c^*} = (r + \gamma\phi^2) + \alpha \int_0^{c^*} dF(\eta) = (r + \gamma\phi^2) + \alpha F(c^*) > 0,$$

(as $c - \eta = 0$ at $\eta = c$.) So

$$\frac{dc^*}{d\phi} > 0$$

Increasing ϕ rapidly improves my chances of getting to trade my good and return to the production island. Investment (in inventory) has a higher return and I am therefore more ready to invest.

(e) Draw a diagram showing the population flows between the islands. Write down the steady-state equations and solve for the population on the trading island as a function of c^*.

The diagram is the same as in the class notes for constant γ except that γ should be replaced with $\gamma\phi^2$. Steady-state equations are:

$$\begin{aligned} \gamma\phi^2 n_T &= \alpha F(c^*) n_P \\ n_T + n_P &= 1 \end{aligned}$$

where n_T is proportion of the population on the trading island and n_P is the proportion of the population on the production island. Thus

$$n_T = \frac{\alpha F(c^*)}{\gamma\phi^2 + \alpha F(c^*)}.$$

8. The preferences of the representative household are

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\left[\ln\left(C_t\right)-\chi\frac{1}{1+\gamma}L_t^{1+\gamma}\right]\right),$$

while the production function is

$$Y_t = Z_t L_t^{1-\alpha}, \quad 0<\alpha<1. \tag{PRF}$$

The log of technology follows an AR(1) process

$$z_t \equiv \ln\left(Z_t\right) = \phi z_{t-1} + \varepsilon_t, \quad 0 \le \phi < 1, \tag{TS}$$

where the exogenous shock ε_t is i.i.d.and zero-mean.

Lump-sum taxes are set to balance the government's budget:

$$T_t = G_t, \tag{BB}$$

where G_t is government spending, which has no effect on production or household utility. Government spending is in turn a function of productivity:

$$G_t = \xi Z_t^{-\nu}, \quad \xi, \nu > 0. \tag{GS}$$

(a) We set about finding the competitive allocation. Recall that the representative producer solves

$$\max_{L_t \ge 0} \Pi_t = Z_t L_t^{1-\alpha} - W_t L_t,$$

so that the first order condition for profit maximization is

$$\left(1-\alpha\right) Z_t L_t^{-\alpha} = \left(1-\alpha\right)\frac{Y_t}{L_t} = W_t, \tag{PM}$$

and profits are

$$\Pi_t = \alpha Y_t.$$

The household's flow budget constraint is

$$C_t + K_{t+1} = \left(1+r\right) K_t + W_t L_t + \Pi_t - T_t, \tag{FBC}$$

so that the first order conditions for utility maximization are

$$\frac{1}{C_t} W_t = \chi L_t^{\gamma}, \tag{LL}$$

$$\frac{1}{C_t} = \beta\left(1+r\right) E_t\left(\frac{1}{C_{t+1}}\right) = E_t\left(\frac{1}{C_{t+1}}\right). \tag{EE}$$

The second F.O.C. is the Euler Equation. Combining equations (PM) and (LL) yields the labor-leisure condition:

$$\left(1-\alpha\right)\frac{1}{C_t} Z_t = \chi L_t^{\gamma+\alpha}. \tag{LL$'$}$$

Combining the consumer's, firm's and government's budget constraints yields the capital accumulation equation:

$$\begin{aligned} K_{t+1} &= \left(1+r\right) K_t + W_t L_t + \Pi_t - T_t - C_t \\ &= \left(1+r\right) K_t + Y_t - G_t - C_t. \end{aligned} \tag{CA}$$

(b) Let lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values. It follows from equation (LL′) that

$$\begin{aligned}
L_t &= \left(\frac{\chi}{1-\alpha}\right)^{\theta} Z_t^{\theta} C_t^{-\theta}, \quad \theta \equiv \frac{1}{\gamma+\alpha} > 0, \\
\Rightarrow \exp\left(\widehat{\ell}_t\right) &= \frac{L_t}{L_{ss}} = \left(\frac{\chi}{1-\alpha}\right)^{\theta} Z_t^{\theta} C_t^{-\theta} \Big/ \left[\left(\frac{\chi}{1-\alpha}\right)^{\theta} Z_{ss}^{\theta} C_{ss}^{-\theta}\right] \\
&= \exp\left(\theta(\widehat{z}_t - \widehat{c}_t)\right).
\end{aligned}$$

Logging both sides yields

$$\widehat{\ell}_t = \theta\left(\widehat{z}_t - \widehat{c}_t\right). \tag{LL″}$$

Proceeding similarly, it follows from equation (PRF′) that

$$\begin{aligned}
\exp\left(\widehat{y}_t\right) &= \exp\left(\widehat{z}_t + (1-\alpha)\theta\left(\widehat{z}_t - \widehat{c}_t\right)\right), \\
\Rightarrow \widehat{y}_t &= (1+\lambda)\,\widehat{z}_t - \lambda\widehat{c}_t, \qquad \text{(PRF″)} \\
\lambda &\equiv \frac{1-\alpha}{\gamma+\alpha}, \quad 0 < \lambda < \theta,
\end{aligned}$$

and it follows from equation (GS) that

$$\widehat{g}_t = -\nu\widehat{z}_t. \tag{GS′}$$

(c) Suppose that $C_{ss}/K_{ss} = \psi$, and that $G_{ss}/K_{ss} = \zeta$, with $(\psi+\zeta) > r$, so that $Y_{ss}/K_{ss} = \psi + \zeta - r$. Implicitly differentiate equation (CA):

$$\begin{aligned}
\exp\left(\ln K_{t+1}\right) &= (1+r)\exp\left(\ln K_t\right) + \exp\left(\ln Y_t\right) - \exp\left(\ln C_t\right) - \exp\left(\ln G_t\right), \\
\Rightarrow \exp\left(\ln K_{t+1}\right) d\ln K_{t+1} &= (1+r)\exp\left(\ln K_t\right) d\ln K_t - \exp\left(\ln Y_t\right) d\ln Y_t \\
&\quad - \exp\left(\ln C_t\right) d\ln C_t. - \exp\left(\ln G_t\right) d\ln G_t.
\end{aligned}$$

so that

$$\begin{aligned}
K_{ss}\widehat{k}_{t+1} &\approx (1+r)\,K_{ss}\widehat{k}_t - Y_{ss}\widehat{y}_t - C_{ss}\widehat{c}_t - G_{ss}\widehat{g}_t, \\
\Rightarrow \widehat{k}_{t+1} &\approx (1+r)\,\widehat{k}_t - \frac{Y_{ss}}{K_{ss}}\widehat{y}_t - \frac{C_{ss}}{K_{ss}}\widehat{c}_t - \frac{G_{ss}}{K_{ss}}\widehat{g}_t,
\end{aligned}$$

which reduces to

$$\widehat{k}_{t+1} \approx (1+r)\,\widehat{k}_t + (\psi+\zeta-r)\,\widehat{y}_t - \psi\widehat{c}_t - \zeta\widehat{g}_t.$$

Then insert (PRF″) and (GS′) to get

$$\widehat{k}_{t+1} \approx (1+r)\,\widehat{k}_t + (\psi+\zeta-r)\left[(1+\lambda)\,\widehat{z}_t - \lambda\widehat{c}_t\right] - \psi\widehat{c}_t - \zeta\left(-\nu\widehat{z}_t\right),$$

so that the log-linear approximation for the capital accumulation equation is

$$\begin{aligned}
\widehat{k}_{t+1} &= (1+r)\widehat{k}_t + \omega_1 z_t - \omega_2\widehat{c}_t, \qquad \text{(CA′)} \\
\omega_1 &= (\psi+\zeta-r)(1+\lambda) + \zeta\nu > 0, \\
\omega_2 &= (\psi+\zeta-r)\lambda + \psi > 0 \\
&= (\psi+\zeta-r)(1+\lambda) - (\zeta-r) = \omega_1 - (\zeta\,(1+\nu) - r).
\end{aligned}$$

(d) One can log-linearize the Euler equation and solve the resulting system (which also includes (TS) and (CA′)) to express consumption as a function of capital and productivity

$$\widehat{c}_t = \eta \widehat{k}_t + \mu z_t. \tag{CF}$$

with

$$\eta = \frac{r}{\omega_2} > 0; \quad \mu = \frac{r\omega_1}{\omega_2 (1 - \phi + r)} > 0.$$

(Take this as given.). Combining equations (CF) and (LL″) yields

$$\widehat{\ell}_t = \theta(1-\mu)\widehat{z}_t - \theta\eta\widehat{k}_t,$$

while combining equations (CF) and (PRF″) produces

$$\widehat{y}_t = [1 + \lambda(1-\mu)]\widehat{z}_t - \lambda\eta\widehat{k}_t.$$

Because $\mu > 1$ is possible (as $\omega_2 < \omega_1$ is possible), labor and output can both be increasing or decreasing in productivity (z). With lump-sum funding, a decrease in government spending has an income effect that leads households to supply less labor. Although an increase in productivity increases labor demand, it also reduces reduces government spending (by equation (GS)), and a reduction in labor is possible. The reduction in labor may in turn be large enough to offset the direct effects of higher productivity (the "1" in the output equation), leading output to fall as well. It is possible, however, that the direct effects are not offset, so that output rises even as labor falls. Note, however, that if labor is increasing in productivity, output will be increasing as well.

(e) If $\mu < 1$, a decrease in productivity will lead to decreases in employment and output. Equation (GS) also shows however, that these will be periods of high government spending. The end result is that times of high government spending will also be times of high unemployment. This does not mean that government spending increases unemployment: our results show that government spending actually increases employment. Concluding that government spending decreases employment reflects a failure to recognize the endogeneity of fiscal policy.

9. **Cash-in-advance in an endowment economy.**

Consider the following economy:

Time: Discrete; infinite horizon.

Demography: Continuum of mass 1 of (representative) households that live for ever.

Preferences: The instantaneous household utility function over consumption, c, is $u(c)$. The function $u(.)$ is twice differentiable, strictly increasing and strictly concave, with $\lim_{c\to 0} u'(c) = \infty$. The discount factor is $\beta \in (0,1)$.

Endowments: In period t, each household receives a quantity e_t of the perishable consumption good (i.e. it is an endowment economy - no production). Each household has an initial stock, H_0 of money.

Institutions: Every period there are markets in which cash is traded for the consumption good.

There is a government that has the power to make transfers of cash, τ_t. (Transfers can be negative.) It increases the money supply at the rate σ so that the per household stock of cash at time t is given by

$$H_t = H_0 (1+\sigma)^t .$$

There is a "cash-in-advance" (CIA) requirement. Households cannot directly consume their own endowment. Instead they have to sell e_t in period t to acquire cash. To consume in period t they have to use money they held at the end of period $t-1$ to buy goods.

(a) Write down the problem faced by a representative household at time 0. (Hint: they will have a budget as well as a CIA constraint.)

The household problem is

$$\max_{\{c_t, m_t^d\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t.} \quad e_t + \frac{m_t}{p_t} = c_t + \frac{m_t^d}{p_t},$$
$$m_t^d + \tau_t = m_{t+1},$$
$$\frac{m_t}{p_t} \geq c_t.$$

(b) Rewrite the problem for the representative household in recursive form for period t.

$$V(m_t, p_t) = \max_{c_t, m_t^d} \left\{ u(c_t) + \beta V(m_t^d + \tau_t, p_{t+1}) \right\}$$

$$\text{s.t.} \quad e_t + \frac{m_t}{p_t} = c_t + \frac{m_t^d}{p_t}, \qquad \frac{m_t}{p_t} \geq c_t.$$

(c) Write down the necessary conditions for a solution to the household's problem (including an envelope condition). (You do not need to solve for the Euler equation.)

Lagrangian:

$$\mathcal{L} = u(c_t) + \beta V(m_t^d + \tau_t, p_{t+1}) + \lambda_t \left(e_t + \frac{m_t}{p_t} - c_t - \frac{m_t^d}{p_t} \right) + \gamma_t \left(\frac{m_t}{p_t} - c_t \right)$$

Necessary conditions comprise the FOC's,

$$c_t \quad : \qquad u'(c_t) = \lambda_t + \gamma_t$$
$$m_t^d \quad : \qquad \beta V_1(m_t^d + \tau_t, p_{t+1}) = \frac{\lambda_t}{p_t}$$

the Kuhn Tucker (complementary slackness) conditions

$$\gamma_t \left(\frac{m_t}{p_t} - c_t \right) = 0, \qquad \gamma_t \geq 0$$

the envelope condition

$$V_1(m_t, p_t) = \frac{\lambda_t + \gamma_t}{p_t}$$

the CIA constraint, the budget constraint, and a transversality condition.

(d) Write down the market clearing conditions and the government budget constraint.
Market clearing implies

$$\begin{aligned} c_t &= e_t \\ H_t &= m_t. \end{aligned}$$

The government budget constraint is

$$\tau_t = H_{t+1} = H_t.$$

(e) Define a competitive equilibrium
A *Competitive equilibrium* is an allocation $\{c_t, m_t\}$ and a sequence of prices $\{p_t\}$ such that given prices, the allocation solves the individual households problem, markets clear and the government budget constraint holds.

(f) Could a social planner who is not subject to the CIA constraint do any better in this economy? Explain.

No. The part of the allocation that matters is $c_t = e_t$ and this is always achieved in equilibrium.

(g) Suppose the cash in advance constraint **binds**. Define $\pi_t = \frac{p_{t+1}}{p_t}$ as the period t gross rate of inflation where p_t is the price of goods in terms of money. Derive an expression for π_t in terms of endowments and the rate of money growth. Interpret your answer.
If it binds, $\frac{m_t}{p_t} = e_t$ so

$$\frac{p_{t+1} m_t}{p_t m_{t+1}} = \frac{e_t}{e_{t+1}} = \pi_t (1 + \sigma).$$

Money has no real effect on this economy. The price level simply adjusts every period in anticipation of the next period endowment and the quantity of money around.

10. The preferences of the representative consumer are

$$E_t\left(\sum_{j=0}^{\infty}\beta^j\left[\ln(c_{t+j})-\chi\frac{1}{1+\gamma}\ell_{t+j}^{1+\gamma}\right]\right),$$
$$0<\beta<1,\quad \gamma\geq 0,\quad \chi>0.$$

The sole source of the single non-storable good is a representative farm that produces the good using labor and.an everlasting tree:

$$y_t=\ell_t^{1-\alpha}.$$

There is a government that levies a tax on labor income. The labor tax rate τ_t follows a Markov process with the stationary transition density $f(\tau',\tau)$. Tax revenues are refunded to consumers through the lump-sum transfer T_t, which the household treats as exogenous.

(a) Let $R_t^{-1}=R^{-1}(\tau_t)$ be price of a one-period risk-free discount bond, $p_t=p(\tau_t)$ be the ex-dividend stock price. Writing the consumer's problem as a Lagrangean, we get

$$\begin{aligned}
&V(x_t,\tau_t)=\\
&\min_{\lambda_t\geq 0}\max_{c_t\geq 0,\ell_t\in[0,1],s_{t+1},b_{t+1}}\ln(c_t)-\chi\frac{1}{1+\gamma}\ell_t^{1+\gamma}\\
&\quad+\lambda_t\left(x_t+(1-\tau_t)w(\tau_t)\ell_t+T_t-c_t-p_t s_{t+1}-R_t^{-1}b_{t+1}\right)\\
&\quad+\beta\int V\left(\left[p_{t+1}(\tau_{t+1})+\pi_{t+1}(\tau_{t+1})\right]s_{t+1}+b_{t+1},\tau_{t+1}\right)\times f(\tau_{t+1},\tau_t)\,d\tau_{t+1},
\end{aligned}$$

where π_t denotes profits from the farm. Note that x_t excludes labor income and transfers. The FOC for an interior solution are:

$$\begin{aligned}
\frac{1}{c_t}&=\lambda_t,\\
\chi\ell_t^{\gamma}&=\lambda_t(1-\tau_t)w_t,\\
\lambda_t p_t&=\beta\int\frac{\partial V[t+1]}{\partial x_{t+1}}\left(p_{t+1}(\tau_{t+1})+\pi_{t+1}(\tau_{t+1})\right)f(\tau_{t+1},\tau_t)\,d\tau_{t+1},\\
\lambda_t R_t^{-1}&=\int\frac{\partial V[t+1]}{\partial x_{t+1}}f(\tau_{t+1},\tau_t)\,d\tau_{t+1}.
\end{aligned}$$

Since (following Benveniste-Scheinkman),

$$\frac{\partial V[t]}{\partial x_t}=\lambda_t,$$

the Euler equations are

$$\begin{aligned}
p_t\frac{1}{c_t}&=\beta E_t\left(\frac{1}{c_{t+1}}(p_{t+1}+\pi_{t+1})\right),\\
R_t^{-1}\frac{1}{c_t}&=\beta E_t\left(\frac{1}{c_{t+1}}\right).
\end{aligned}$$

The labor allocation condition is

$$\chi \ell_t^\gamma = (1-\tau_t)\frac{1}{c_t} w_t.$$

(b) Next, we find the equilibrium allocations.

1. The representative farm solves:

$$\max_{\ell_t \geq 0} \pi_t = \ell_t^{1-\alpha} - w_t \ell_t.$$

The first-order condition for an interior solution is

$$w_t = (1-\alpha)\,\ell_t^{-\alpha},$$

while profits are

$$\ell_t^{1-\alpha} - \ell_t\,(1-\alpha)\,\ell_t^{-\alpha} = \alpha y_t.$$

2. Clearing in the market for goods implies that

$$y_t = c_t.$$

It follows that

$$w_t = (1-\alpha)\,\frac{y_t}{\ell_t} = (1-\alpha)\,\frac{c_t}{\ell_t}.$$

so that the labor allocation condition becomes

$$\begin{aligned} \chi \ell_t^\gamma &= (1-\tau_t)\,(1-\alpha)\,\frac{1}{\ell_t}, \\ \Rightarrow \ell_t &= \left[\left(\frac{1-\alpha}{\chi}\right)(1-\tau_t)\right]^{1/(1+\gamma)} \\ &= \ell\,(\tau_t)\,. \end{aligned}$$

3. We now have

$$\begin{aligned} y_t &= \ell\,(\tau_t)^{1-\alpha} = \left[\left(\frac{1-\alpha}{\chi}\right)(1-\tau_t)\right]^{(1-\alpha)/(1+\gamma)} \\ &= y\,(\tau_t)\,. \end{aligned}$$

(c) In equilibrium $c_t = y\,(\tau_t)$ and $\pi_t = \alpha y\,(\tau_t)$. With these changes, the Euler equation for stocks becomes

$$p_t \frac{1}{y\,(\tau_t)} = \beta E_t \left(\frac{1}{y\,(\tau_{t+1})}\,(p_{t+1} + \alpha y\,(\tau_{t+1}))\right),$$

or

$$p_t y\,(\tau_t)^{-1} = \beta E_t \left(p_{t+1} y\,(\tau_{t+1})^{-1} + \alpha\right),$$

Since $0 < \beta < 1$, it makes sense to solve this equation forward:

$$E_t \left(\left(1 - \beta L^{-1}\right) p_t y\,(\tau_t)^{-1}\right) = \alpha\beta,$$

so that

$$\begin{aligned} p_t y(\tau_t)^{-1} &= \frac{1}{1-\beta L^{-1}} \alpha\beta + b_t \\ &= \alpha \frac{\beta}{1-\beta} + b_t, \end{aligned}$$

with the "bubble term" b_t obeying

$$E_t(b_{t+1}) = \beta^{-1} b_t.$$

We now require that

$$\lim_{J\to\infty} E_t\left(\beta^J p_{t+J} \frac{1}{y(\tau_{t+J})}\right) = 0, \quad \forall t. \tag{TVC}$$

Equation (TVC) will be satisfied only if $b_t = 0$ and the price of a stock is

$$\begin{aligned} p_t &= \alpha \frac{\beta}{1-\beta} y(\tau_t) \\ &= \alpha \frac{\beta}{1-\beta} \left[\left(\frac{1-\alpha}{\chi}\right)(1-\tau_t)\right]^{(1-\alpha)/(1+\gamma)} \\ &= p(\tau_t). \end{aligned}$$

(d) Inserting the equilibrium allocation into the Euler equation for bonds yields

$$\begin{aligned} R^{-1}(\tau_t) &= y(\tau_t)\beta E_t\left(\frac{1}{y(\tau_{t+1})}\right) \\ &= y(\tau_t)\beta E_t\left(\left[\left(\frac{1-\alpha}{\chi}\right)(1-\tau_{t+1})\right]^{-(1-\alpha)/(1+\gamma)}\right). \end{aligned}$$

Note that

$$\begin{aligned} &\frac{\partial}{\partial\tau_{t+1}}\left[\left(\frac{1-\alpha}{\chi}\right)(1-\tau_{t+1})\right]^{-(1-\alpha)/(1+\gamma)} \\ &= -\frac{1-\alpha}{1+\gamma}\left[\left(\frac{1-\alpha}{\chi}\right)(1-\tau_{t+1})\right]^{-(1-\alpha)/(1+\gamma)-1}\left(-\frac{1-\alpha}{\chi}\right) \\ &= \frac{(1-\alpha)^2}{\chi(1+\gamma)}\left[\left(\frac{1-\alpha}{\chi}\right)(1-\tau_{t+1})\right]^{-(1-\alpha)/(1+\gamma)-1}. \end{aligned}$$

This derivative is positive and, because its exponent is negative, is increasing in τ_{t+1}. This means that $y(\tau_{t+1})^{-1}$ is a convex function of τ_{t+1}. It follows from Jensen's inequality that $E_t\left(y(\tau_{t+1})^{-1}\right)$ is increasing in the conditional variance of τ_{t+1}. What we are seeing is akin to precautionary saving. Because marginal utility is convex in τ_{t+1}, an increase in the conditional variance of τ_{t+1} increases the expected marginal utility of future consumption. In this economy, where equilibrium consumption must equal output, equilibrium bond holdings do not increase in response to an increase in expected marginal utility. Rather, the price of discount bonds rises (the return falls) until consumers are content to consume all output.